Statistical Wireless Channel Model

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Abstract

In this article, we discuss the statistical model of the wireless channel in reasonable detail. We start with the most intuitive model of the time-varying channel and derive statistical parameters which are used to characterise real-life channels. We derive both the first and second order statistics and indicate their physical significance.

1 Introduction

The wireless channel is a widely used but rarely understood element in the design of communication systems today. One can often find people working with various parameters of a wireless channel without an understanding of the basics. The author has till date not found any good tutorial which explains all the basic elements of the channel in reasonable detail. Most books only touch upon some aspects of the channel and ignore the rest. Moreover, there is a large variation in the treatment of the topic across books which thoroughly confuses the reader.

In this article we introduce the channel as a two-dimensional random process and derive the various parameters used to characterise it. The focus of this article is to derive the different statistical parameters in an intuitive manner. The aim is to bridge the gap between the statistical model and the physical understanding of the channel. Therefore, we skip the mathematical derivations and focus on the physical significance of the different observable parameters that can be derived from the statistical model.

2 Complex Baseband Channel Model

The wireless channel is incomparably more hostile than the AWGN channel which is usually the channel model assumed for most wired communication systems. A typical wireless channel is a time-varying system in which the parameters are random and liable to change with time.

The channel is modeled as a linear time-varying random filter whose impulse response can be expressed as a set of complex (baseband) channel gains at a given time t and a given delay τ. This defines a two-dimensional random process $h(t, \tau)$. Thus, the (baseband) output of the channel is defined by the convolution integral

$$s_o(t) = \int_0^\infty h(t - \tau, \tau) s(t - \tau) d\tau \qquad (1)$$

where

$s(t)$	:	The input signal
$h(t, \tau)$	:	The complex channel gain (a random process)
τ	:	The propagation delay
$s_o(t)$	:	The output signal

The first order distribution of the complex channel gain $h(t, \tau)$ is typically modeled by a complex Gaussian random variable. Then, at any time t, the absolute value $|h(t, \tau)|$ is a Rayleigh (when $E[h(t, \tau)] = 0$) or Ricean (when $E[h(t, \tau)] \neq 0$) distributed random variable.

The time-varying transfer function of the channel is defined as the Fourier transform of $h(t, \tau)$ with respect to the delay variable τ,

$$H(t, f) = \int_{-\infty}^{\infty} h(t, \tau) e^{-j2\pi f \tau} d\tau \qquad (2)$$

where f denotes the frequency variable. This time-varying transfer function can be viewed as a frequency transmission characteristic of the channel.

3 Channel Autocorrelation Functions

3.1 Autocorrelation of the Impulse Response

Since the channel coefficients $h(t, \tau)$ define a random process, we now look at the autocorrelation properties of this process. The general autocorrelation function can be written as

$$R_{hh}(t_1, t_2, \tau_1, \tau_2) = E[h(t_1, \tau_1) h^*(t_2, \tau_2)] \qquad (3)$$

We make the following assumptions about the channel statistics

- The channel is assumed to be Wide Sense Stationary (WSS) in the time-domain, *i.e.* the autocorrelation function depends only on the difference between the time instants rather than on the actual time instants.

- The channel exhibits *uncorrelated scattering*, *i.e.* the contribution from the scatterers with different delays is uncorrelated.

With these assumptions, we can write (3) as

$$E\left[h(t+\Delta t,\tau_1)h^*(t,\tau_2)\right] = R_{hh}(\Delta t,\tau_1)\delta(\tau_1-\tau_2) \tag{4}$$

3.2 Autocorrelation of the Transfer Function

Taking the autocorrelation of the time-varying transfer function of the channel (2), we get

$$R_{HH}(\Delta t, f_1, f_2) = E\left[H(t+\Delta t, f_1)H^*(t,f_2)\right] \tag{5}$$

It can be shown that the assumption of uncorrelated scattering implies that the autocorrelation function of the channel transfer function is only a function of the frequency difference [1]. Thus,

$$E\left[H(t+\Delta t, f)H^*(t, f+\Delta f)\right] = R_{HH}(\Delta t, \Delta f) \tag{6}$$

3.3 Autocorrelation with respect to the Delay Variable τ

3.3.1 Power Delay Profile

Evaluating the autocorrelation of the channel impulse reponse (4) at $\Delta t = 0$, we get

$$R_{hh}(\tau) \equiv R_{hh}(0,\tau) = R_{hh}(\Delta t,\tau)|_{\Delta t=0} \tag{7}$$

This is called the *multipath intensity profile* or the *power delay profile* of the channel. This gives the average power output of the channel as a function of the delay τ. The range of values over which $R_{hh}(\tau)$ is non-zero is called the *multi-path delay spread of the channel* and is denoted by T_m. This is simply the maximum propagation delay observed in the channel which in turn characterises the time dispersiveness (Inter Symbol Interference (ISI)) of the channel.

3.3.2 Power Delay Spectrum

Evaluating the autocorrelation of the channel transfer function (6) at $\Delta t = 0$, we get

$$R_{HH}(\Delta f) \equiv R_{HH}(0,\Delta f) = R_{HH}(\Delta t,\Delta f)|_{\Delta t=0} \tag{8}$$

This provides us with a measure of the frequency coherence of the channel and the bandwidth $(\Delta f)_c$ of $R_{HH}(f)$ is called the *coherence bandwidth* of the channel. It is a measure of the frequency selectivity of the channel during transmission and indicates the frequency width over which the channel can be considered to be flat. Thus, two sinusoids with frequency separation greater than $(\Delta f)_c$ undergo different attenuations in the channel.

If $(\Delta f)_c$ is small in comparison to the bandwidth of the transmitted signal, the channel is said to be *frequency selective*. On the other hand, if $(\Delta f)_c$ is large in comparison to the transmitted signal bandwidth, the channel is said to be *frequency non-selective* or *frequency flat*.

Also, it can be easily shown that

$$R_{hh}(\tau) \overset{\mathcal{F}}{\longleftrightarrow} R_{HH}(f) \tag{9}$$

i.e. $R_{hh}(\tau)$ and $R_{HH}(f)$ form a Fourier transform pair. As a result of this relationship, the reciprocal of the multi-path delay spread is a measure of the coherence bandwidth of the channel. That is,

$$(\Delta f)_c \approx \frac{1}{T_m} \tag{10}$$

3.4 Autocorrelation with respect to the Time Variable t

The autocorrelation with respect to the time variable t denotes the behaviour of the random process at each delay τ, *i.e.* for a given $\tau = \tau_0$, the autocorrelation with respect to t characterises the random process $h(t, \tau_0)$.

3.4.1 Scattering Function

Taking the Fourier transform (with respect to Δt) of the autocorrelation of the channel impulse reponse (4) at $\tau = \tau_0$, we get

$$S(\lambda, \tau_0) = \int_{-\infty}^{\infty} R_{hh}(\Delta t, \tau_0) e^{-j2\pi\lambda\Delta t} d(\Delta t) \tag{11}$$

This is called as the *scattering function* of the channel. It gives us the power spectral density of the random process which defines the channel coefficient at a particular delay τ_0.

3.4.2 Doppler Spectrum

Integrating the scattering function over all possible delays, we get

$$S(\lambda) = \int_{-\infty}^{\infty} S(\lambda, \tau) d\tau \tag{12}$$

In other words, $S(\lambda)$ is the Fourier transform of (11) with respect to τ_0 evaluated at $f = 0$, *i.e.*

$$S(\lambda) \equiv S(\lambda, f)|_{f=0} = \left(\int_{-\infty}^{\infty} S(\lambda, \tau) e^{-j2\pi f \tau} d\tau \right)_{f=0} \tag{13}$$

$S(\lambda)$ is called the *Doppler spectrum* of the channel and is a measure of the frequency dispersiveness of the channel. The bandwidth of $S(\lambda)$ is also referred to as the *Doppler spread* of the channel. The reciprocal of the Doppler spread is a measure of the coherence time of the channel which denotes the rate of varations in the channel with respect to time. Smaller the coherence time $(\Delta t)_c$, faster will be the time variations in the channel. That is

$$(\Delta t)_c \approx \frac{1}{B_d} \tag{14}$$

Thus, if $(\Delta t)_c$ is small in comparison to the transmission time interval, the channel is said to be *fast fading*. On the other hand, if $(\Delta t)_c$ is large in comparison to the transmission time interval, the channel is said to be *slow fading*.

3.5 Discrete-Time Discrete-Delay Channel Model

While designing digital wireless communication systems, discrete-time discrete-delay models of the wireless channel are used. The discrete-time discrete-delay equivalents of all of the above functions can be derived analogous to the above analysis.

Examples of the characterisation of practical wireless channels can be found in [2, 3]. The models in these references define the number of multi-paths and the relative average power received for each multi-path delay. They also define the Doppler spectrum which broadly characterises the time variations of the channel.

References

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