Minimum Probability of Error Based Prefiltering for DS-CDMA Systems

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Abstract— **In order to reduce the complexity of the mobile receiver, we develop two prefilter models using a linear FIR prefilter for minimizing the Probability of Error (MPOE) in Multi-user transmission scenario. We consider downlink transmission. In the first system model we have single common prefilter for all users at the base station transmitter and in the second prefilter design model we consider individual prefilter for each and every user. We assume complete knowledge of the channel at the base station. In order to fully utilise the knowledge available at the transmitter, the filter weights are computed conditioned on the transmitted bit vector sequence. This also makes the computation of the optimal coefficients linear in the number of users as opposed to the exponential complexity otherwise. Weights of FIR prefilter are computed by minimizing the conditional Probability of Error and the Mean Square Error (MMSE). Simulation results are provided to illustrate the performance of the system model.**

I. Introduction

DS-CDMA suffers from multiple access interference (MAI) which is caused by interference among the users' signals in multipath propagation. MAI mitigation has been a challenging research topic since the very beginning of studies on CDMA. Yet, the frequently considered approach of performing multi-user detection at the mobile receiver is quite unattractive for the downlink, because it entails an increase of complexity and power consumption at the mobile terminals. The solution lies in, transferring the work load to the base station transmitter in the form of prefiltering. In this paper, we explore a prefiltering scheme at the transmitter [1] which can allow for considerably simplified receiver structures. It is evident that prefiltering will only be useful if the channel variation timescales are relatively slower than the time taken for the channel to be estimated at the transmitter. The downlink channel can be estimated at the transmitter by using some feedback from the receiver [2][3]. Alternately, for Time Division Duplexing (TDD) based systems, estimates of the uplink channel can be used as the channel parameters for the downlink channel as well if the time interval between switching from uplink to downlink is small enough. We consider two approaches for prefiltering: the first one considers the common prefilter for all users as shown in Fig. 1. Since the prefiltering is done jointly for all users in this case, the performance of joint prefiltering will not be as desired. To improve the performance further in the second case we consider a system which has individual prefilter for each user data at the base station transmitter as shown in Fig. 2. The standard single user receiver (conventional matched filter detector) is used in our both the models. MMSE has traditionally been used as the optimization criterion in the design of most wireless systems. Nevertheless we believe that since the model is being developed for a digital communications system, the optimum filter will be the one which minimizes the probability of symbol error at the receiver. MPOE based receivers usually tend to be computationally expensive but since we are assuming enough computational resources at the base station, using MPOE instead of MMSE as the optimisation criterion can be justified. Moreover, by conditioning the filter weights on the transmitted bits, one can design an MPOE prefilter with linear complexity. The novelty and the contribution of this paper lies in developing the MPOE based joint and individual prefiltering.

II. Related Work

Significant amount of research work has been carried out in the area of prefiltering over the last few years. But almost all the work have been directed toward the design of MMSE based prefiltering wherein the optimization criterion is minimizing the mean squared error between the transmitted and the demodulated waveform at the receiver. Vojcic and Jang in [1] and Reynolds *et al*, in [4] consider a synchronous multiuser CDMA system and design the prefilter using the MMSE criterion. The analysis in both the papers is assuming zero Inter-Symbol-Interference (ISI). Dua and Desai in [5] first proposed the MPOE optimization method. Later Dua *et al* in [6], Sood *et al* in [7] and Mohit *et al* in [8] extended it to different scenarios. In [5, 6, 7, 8] it was established that MPOE optimization has better performance than MMSE optimization, moreover an linear complexity with respect to number of users also proposed. In [9] Ding *et al* proposed a precoder based on minimum BER method for Zero-Forcing (ZF) equalizer at the receiver. This receiver needs training or, both channel and precoding filter knowledge at the receiver. This will require a relatively complex receiver which may not be desirable. The primary objective of prefiltering is to simplify the receiver structure, hence we work with a conventional single-user detector at the receiver. Georgoulis *et al* in [10, 11] use the simple matched filter receiver and consider a general channel model with ISI while optimizing the filter on the basis of the MMSE criterion. In this paper MPOE based prefiltering with general channel model with MAI (due to multiple users) has been proposed.

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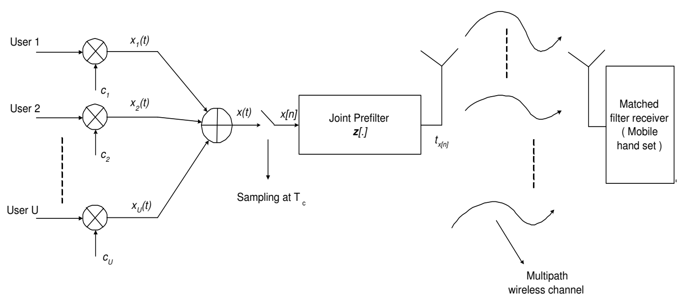

Fig. 1. DS-CDMA channel model for an asynchronous multipath channel with joint transmitter prefiltering

III. Signal Model

Consider a DS-CDMA system with K number of users as shown in Fig 1. The k^{th} user transmits $b_k(i)$ bit in i^{th} interval and the length of signaling interval for each user is T [13, 14]. User k is assigned a spreading waveform $c_k(.)$ which is supported in [0, T] and is normalised to 1. Let $\mathbf{s}_k = [s_{k1}, s_{k2}, \ldots, s_{kN}]$ denote the corresponding spreading sequence then,

$$c_k(t) = \frac{1}{\sqrt{N}} \sum_{n=1}^{N} s_{kn} rect[t - (n-1)T_c]$$

where, $rect(t)$ is a rectangular waveform with unit amplitude in [0, T] and N is the processing gain of the system. The baseband signal of the k^{th} user in the i^{th} bit interval can now be expressed as

$$x_k(t) = A_k b_k(i) c_k(t - iT), \qquad iT \le t \le (i+1)T$$

Let $x(t)$ denote the transmitted signal by the base station in i^{th} interval,

$$x(t) = \sum_{u=1}^{K} A_u b_u(i) c_u(t - iT_{bit}), \qquad iT \le t \le (i+1)T$$

Assume that $x(t)$ is sampled $(x[n])$ at the chip period (T_c) to process the signal in the prefilter

$$x[n] = x[nT_c] = \frac{1}{\sqrt{N}} \sum_{u=1}^{K} A_u b_u(i) \tilde{s}_u[n] \qquad (1)$$

where $\tilde{s}_u[n] = \ldots, s_{u0},\ s_{u1}, \ldots, s_{uN-1},\ s_{u0}, \ldots$ We assume that the prefilter for a particular bit period i is an FIR filter $(z[.][i])$ of length L_z. The idea is to compute these filter coefficients using MPOE criteria. The channel is modeled as an FIR filter of length L_h, and the channel for u^{th} user at i^{th} bit interval is denoted as $h_u[.][i]$.

For a narrowband flat-fading channel, a general entry of the channel FIR filter is denoted by

$$h_u[l][n] = \alpha + j\beta = \sqrt{\alpha^2 + \beta^2} e^{-jtan^{-1}(\frac{\alpha}{\beta})} = \mid h_u[l][n] \mid e^{j\phi_{ij}}$$

where $\mid h_u[l][n] \mid$ represents the fading envelope of the l^{th} coefficient of narrowband FIR channel at time instant n between u^{th} user and the mobile receiver. If both α and β are zero-mean, independent, and normally distributed random variables, $h_u[l][n]$ will be a zero-mean, complex, normally distributed random variable with a fading envelope $\mid h_u[l][n] \mid$ following a Rayleigh distribution. For non line of sight transmission, it is common to assume all the entries of the channel matrix as (possibly correlated) complex random variables with Rayleigh distributed envelopes [15]. The noise is also assumed to be zero mean additive white Gaussian (AWGN). The received signal is

$$r_u[n] = h_u[.][n] * x[n] * z[.][n] + \eta_u[n]$$

where * denotes convolution operation. Converting $r_u[n]$ into a parallel stream of N samples each for processing, we get

$$\mathbf{r}_u[n] = [r_u[iN], \ldots, r_u[iN + N - 1]]^T$$

Receiver is a simple matched filter matched with user's PN sequence. The received signal corresponding to u^{th} user after matched filtering is

$$y_u[i] = \mathbf{s}_u^T \mathbf{r}_u[i] = \sum_{k=0}^{N-1} s_{uk} r_u[iN + k]$$

$$\begin{aligned} y_u[i] = \Bigg[\sum_{k=0}^{N-1} s_{uk} \sum_{m=0}^{L_h^u - 1} h_u[m][j_1] \sum_{l=0}^{L_z^u - 1} z[l][j_2]. \\ .\sum_{u=1}^{K} A_u b_u[j_2] \tilde{s}_u[iN + k - m - l] \Bigg] \\ + \sum_{k=0}^{N-1} s_{uk} \eta_u[iN + k] \end{aligned} \qquad (2)$$

where

$$j_1 = \left\lfloor \frac{iN + k - m}{N} \right\rfloor, \quad j_2 = \left\lfloor \frac{iN + k - m - l}{N} \right\rfloor \qquad (3)$$

$\sum_m$ gives the ISI term while $\sum_u$ is the MAI component. The decision statistic of the u^{th} user is $\Re([y_u[i]])$. We will now derive the MPOE and MMSE based algorithms for the demodulation at the u^{th} user with the above decision statistic $\Re([y_u[i]])$ and using zero as the threshold. Note that zero may not be the ideal threshold under the given scenario but since the receiver does not have any knowledge about the channel characteristics, it cannot determine the optimal threshold. We compute the probability of error at the receiver of the u^{th} user for the i^{th} bit. Since the transmitted bit is known at the transmitter where all processing is being carried out, we use conditional probability of error conditioned on the sequence of transmitted bit vector for all users $(\mathbf{B}[i] = \mathbf{b}[i], \mathbf{b}[i-1], ...,)$. Then, the conditional mean and variance of the of the decision statistics of the received signal $(\Re(y_u^R))$ are given as

$$\begin{aligned} \mu_{y_u^R|B[i]}[i] = E(y_u^R|B[i]) = \Re \Bigg[\sum_{k=0}^{N-1} s_{uk} \sum_{m=0}^{L_h^u - 1} h_u[m][j_1]. \\ .\sum_{l=0}^{L_Z - 1} z[l][j_2] \left(\sum_{u=1}^{K} A_u b_u[j_2] \tilde{s}_u[iN + k - m - l] \right) \Bigg] \end{aligned} \qquad (4)$$

here $E(\eta_u[.]) = 0$ and j_1, j_2 are given in (3).

$$\sigma^2_{y_u^R|B[i]}[i] = var\Bigg(\Re\Bigg[\sum_{k=0}^{N-1} s_{uk} \sum_{m=0}^{L_h^u-1} h_u[m][j_1] \sum_{l=0}^{L_Z-1} z[l][j_2]. \cdot\Bigg(\sum_{u=1}^{K} A_u b_u[j_2]\tilde{s}_u[iN+k-m-l]\Bigg) + \sum_{k=0}^{N-1} s_{uk}\eta_u[iN+k]\Bigg]\Bigg) = N\frac{\sigma^2}{2}$$

where for simplicity the variance of the channel noise σ^2 is assumed to be constant for all users. Now the conditional probability of error is

$$P_{E|B[i]}[i] = Q\left(\frac{b_u[i]\mu_{y_u^R|B[i]}[i]}{\sigma\sqrt{N/2}}\right) \quad (5)$$

IV. Proposed joint prefiltering algorithm

A. MPOE based prefilter

In joint prefiltering we have one common prefilter and we would like to minimize the joint probability of error for all users, namely,

$$P_{Ej}[i] = 1 - P[y_1^R \in \alpha_1, y_2^R \in \alpha_2, \ldots, y_K^R \in \alpha_K]$$

where α_is are decision regions for symbol detection. Since both the noise vectors for all users are independent of each other, the joint probability of error becomes

$$P_{Ej}[i] = 1 - P[y_1^R \in \alpha_1]P[y_2^R \in \alpha_2]\ldots P[y_K^R \in \alpha_K]$$

The decision region for BPSK constellation, α_u, for any user u, is given by $(0,\infty)$ when $b_u[i] = +1$ and $(-\infty, 0)$ when $b_u[i] = -1$.
By using the above equations, P_{Ej} can be written in closed form as

$$P_{Ej}[i] = 1 - \prod_{u=1}^{K} Q\left(-\frac{b_u\mu_{y_u^R}}{\sigma\sqrt{N/2}}\right)$$

Thus the MPOE optimization problem now becomes

$$min_{z[.][i]}P_{Ej}[i]$$

That is, the filter coefficients ($\mathbf{z}[.][i]$) of length L_z for each bit interval i is calculated by minimizing the above formulated probability of error. A stochastic gradient descent approach can now be used to minimize the joint probability of error expression with respect to the prefilter coefficients. In gradient descent, the prefilter coefficients are updated according to the rule

$$\mathbf{z}[.][i+1] = \mathbf{z}[.][i] - \mu\frac{\partial P_{Ej}}{\partial \mathbf{z}[.][i]}$$

where μ is an appropriately chosen step-size parameter, and in general could be adaptive.

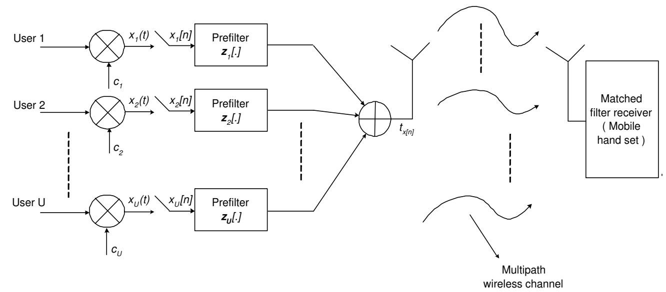

Fig. 2. DS-CDMA channel model for an asynchronous multipath channel with individual transmitter prefiltering

B. MMSE based prefilter

At each instant of time i, we can write the sufficient statistic of the u^{th} user as the real part of $y_u[i]$. Thus, at every time instant we get a vector $\mathbf{y}^R[i]$ of decision statistics for all users. Therefore the cost function in case of MMSE based algorithm can be written as

$$\xi^2_{J|B[i]}[i] = E\Big[\|\mathbf{y}^R[i] - \mathbf{b}[i]\|^2|B[i]\Big] = \sum_{u=1}^{K} E\Big[(y_u^R)^2 + b_u^2 - 2y_u^R b_u)|B\Big] = \sum_{u=1}^{K}\Big[((\tilde{y}_u^R)^2) + \sigma^2\frac{N}{2} + 1 - 2b_u(\tilde{y}_u^R)\Big]$$

where J denotes the joint norm for all the users, $\mathbf{b}[i]|B[i]$ is the transmitted bit vector from all users at time instant i, $\tilde{y}_u^R$ is given in (4). Now the filter weights are calculated from

$$min_{z[.][i]}P_{Ej}[i]$$

We follow an exactly similar optimisation framework as for the MPOE algorithm with the expectations being conditioned on the transmitted bit vector sequence followed by normalisation of the transmitter prefilter coefficients. We compute the numerical gradient of ξ^2 with respect to z[.] and optimise the filter weights. The simulation results are presented in Section VI.

V. Proposed individual prefiltering algorithm

In this model the data of user u after being spread is prefiltered by an FIR filter of length L_z with a discrete time impulse response $z_u[.]$ as shown in Figure 2. The resulting modified signals are summed to form the final transmitted signal. Since the filters are applied at the chip level, the derivation of the algorithms are restricted to consider only a few symbols. Individual prefilters have also the advantage of not modifying the original CDMA structure directly as it is a simple addition of an array of FIR filters to the existing base station transmission system. The prefilters $z_u[.]$, u = $1, \ldots, K$ are designed such that the probability of error meant for that particular user is minimum at the receiver. Each users prefilter is designed individually by taking into account the channel information and the transmit codes of that particular user. The signal for user u in base station is

$$x_u(t) = A_u b_u(i) c_u(t - iT_{bit}), \qquad iT \le t \le (i+1)T$$

This signal is sampled at chip level as explained in equation (1) and will be processed through a prefilter $z_u[.]$. The transmitted signal at time instant n, corresponding to user u after prefiltering is given by

$$x_u[n] * z_u[.][n]$$

Now the total transmitted signal $t_x[n]$ at base station is given by

$$t_x[n] = \sum_{u=1}^{K} x_u[n] * z_u[.][n]$$

where * is the convolution operator. If we carry out the same formulation as in equation (2), the received signal for user u after matched filtering is

$$y_u[i] = \Bigg[\sum_{k=0}^{N-1} s_{uk} \sum_{m=0}^{L_h^u-1} h_u[m][j_1] \sum_{u=1}^{K} \sum_{l=0}^{L_z^u-1} . \\ .z_u[l][j_2] A_u b_u[j_2] \tilde{s}_u[iN+k-m-l] \Bigg] \\ + \sum_{k=0}^{N-1} s_{uk} \eta_u[iN+k]$$

A. MPOE based prefilter

The individual probability of error is minimized for MPOE based individual prefiltering algorithm. The probability of error for a user u at the mobile receiver $P^u_{E|B[i]}[i]$ can be formulated as given in equation (5), except the fact that prefilter is different for each users.

$$P^u_{E|B[i]}[i] = Q\left(\frac{b_u[i] \mu_{y_u^R|B[i]}[i]}{\sigma\sqrt{N/2}} \right)$$

where $\mu_{y_u^R|B[i]}[i]$ is given by

$$\mu_{y_u^R|B[i]}[i] = E(y_u^R|B[i]) = \Re \Bigg[\sum_{k=0}^{N-1} s_{uk} \sum_{m=0}^{L_h^u-1} \sum_{u=1}^{U} h_u[m][j_1]. \\ . \sum_{l=0}^{L_Z-1} z_u[l][j_2] A_u b_u[j_2] \tilde{s}_u[iN+k-m-l] \Bigg] \quad (6)$$

Now the filter weights of length L_z at time instant $i+1$ for user u is calculated as follows using stochastic gradient search method

$$\mathbf{z}_u[.][i+1] = \mathbf{z}_u[.][i] - \mu \frac{\partial P_E^u}{\partial \mathbf{z}_u[.][i]}$$

where μ is the step size parameter.

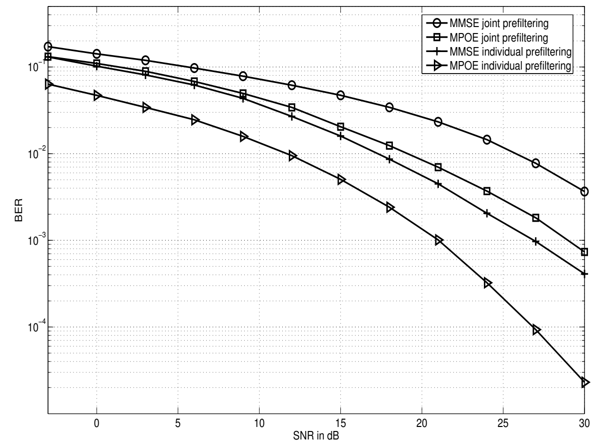

Fig. 3. Performance of MPOE and MMSE transmitter prefiltering with 4 users

B. MMSE based prefilter

The function to be optimized for the case of MMSE based individual prefiltering can be written as

$$\xi^2_{u|B[i]}[i] = E\left[||y_u^R[i] - b_u[i]|B[i]||^2 \right] \\ = E\left[(y_u^R)^2 + b_u^2 - 2y_u^R b_u)|B \right] \\ = (\tilde{y}_u^R)^2 + \sigma^2 \frac{N}{2} + 1 - 2b_u(\tilde{y}_u^R)$$

where $\tilde{y}_u^R$ is same as given in equation (6). Now the individual MMSE prefilter coefficients are calculated using gradient search by minimizing $z_u[.]$ as follows

$$min_{z_u[.][i]} P_{Ej}[i]$$

We follow the same procedure as of joint prefiltering algorithm.

VI. SIMULATIONS AND RESULTS

Extensive simulations were carried out to calculate the filter coefficients and the corresponding bit error rate for various SNRs for the proposed models with MPOE and MMSE prefilters. The BPSK constellation data was generated with equal probability for bits +1 and -1. The processing gain, N, was assumed to be 128 and the simulations were done with 4 and 8 users. Channel was assumed to be complex Gaussian with both real and imaginary part follows independent Gaussian distribution with σ value as 0.1655 and mean as 0.5. The channel coefficients were assumed to be constant over one bit interval. Bit error rate (BER) of 50 such channels were

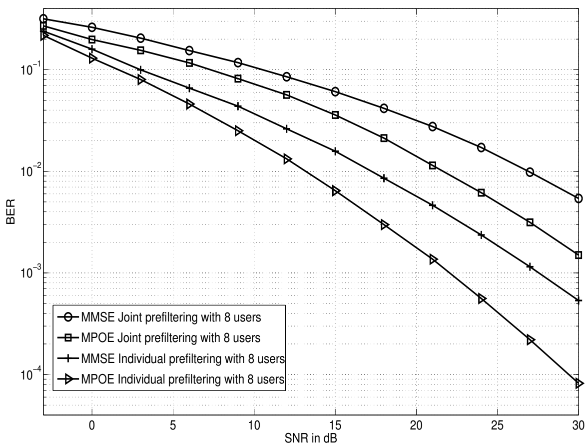

Fig. 4. Performance of MPOE and MMSE transmitter prefiltering with 8 users

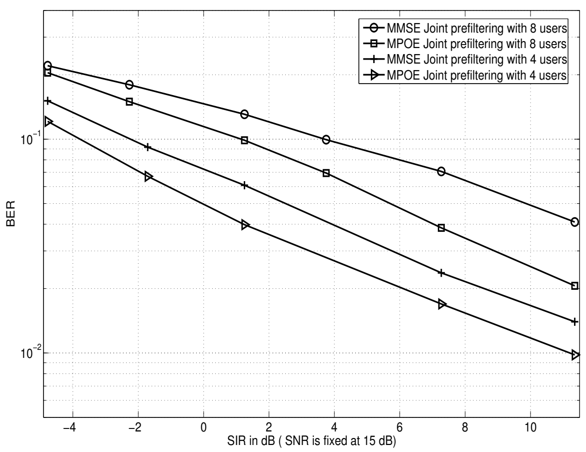

Fig. 5. Performance of MPOE and MMSE joint prefiltering for various SIR and fixed SNR of 15 dB

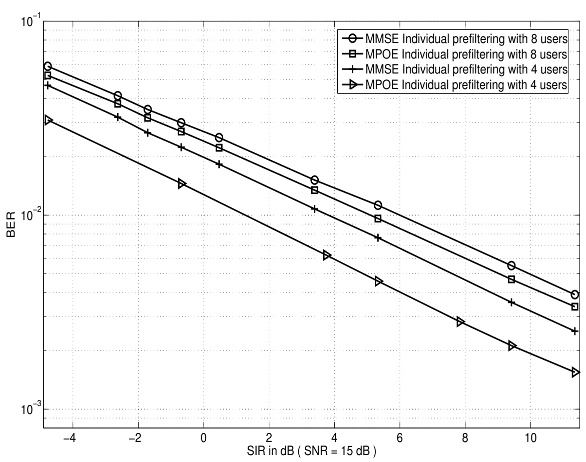

Fig. 6. Performance of MPOE and MMSE individual prefiltering for various SIR and fixed SNR of 15 dB

averaged for each SNR. Results for channel length, $L_h = 4$ and prefilter length, $L_z = 5$ for varying SNRs are shown in Fig. 3 and Fig. 4. We assume the spreading codes have a correlation coefficient $\rho = 0.23$. A higher value for ρ has been chosen to show the effect of MAI. Fig. 3 shows the BER performance for the case of 4 users. The plots validate the better performance of individual prefiltering over joint prefiltering and of MPOE prefiltering over MMSE prefiltering. The performance of MPOE based individual prefiltering is about 8 dB better than that of MPOE based joint prefiltering.

To explore the effect of MAI further, the number of users was increased to 8 and performance of proposed models with same correlation coefficient is shown in Fig. 4. As expected BER of all algorithms varies inversely with the number of users. The improvement due to individual prefiltering is still substantial.

The performance of the proposed model for different ρ has been investigated. The ρ was varied from 0.15 to 1 in 8 steps and SIR was calculated for a fixed SNR of 15 dB for both the models. Fig. 5 and Fig. 6 show the BER performance for various SIR and fixed SNR for individual and joint prefiltering models respectively. From Fig. 5 and Fig. 6 we can observe that as the ρ increases the performance reduces due to MAI. Moreover for a fixed SIR and SNR the individual prefiltering model BER is always lesser than that of joint prefiltering model. The particular choice of ρ is the compromise to be chosen between system capacity and performance since an increase in ρ increases system capacity but decreases the performance.

VII. CONCLUSION

We have proposed two system models with MPOE prefiltering technique for DS-CDMA systems. Simulation results show that MPOE prefilter is superior to that of MMSE in terms of bit error rate. It is also evident that individual prefiltering out performs joint prefiltering.

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