Multi-user Detection

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Abstract

In a CDMA system, many users simultaneously transmit across the entire frequency band and users' data is separated on the basis of their signature waveforms. It is logical that the user signature waveforms be mutually orthogonal so as to avoid interference among different users. In this article, we briefly touch upon the reason for non-orthogonal signature waveforms being deployed in CDMA based cellular systems. The conventional matched filter detector is introduced and its drawbacks are discussed. We next discuss the optimum multi-user detector proposed by Verdú [1]. Lastly, we will briefly touch upon some sub-optimal approaches to multi-user detection that have been proposed in literature.

1 Introduction

Code Division Multiple Access (CDMA) is a widely used technique for multiple access communication in wireless systems. It differs from the classical Time Division Multiple Access (TDMA) and Frequency Division Multiple Access (FDMA) in the sense that all users transmit across the entire frequency band (unlike FDMA) and many users can transmit simultaneously (unlike TDMA). The user data is separated on the basis of signature waveforms assigned to each user. These waveforms should be mutually orthogonal to each other so as to eliminate any interference among different users' data.

It can be shown that the number of orthogonal waveforms that can be designed within a given bandwidth (W) and time duration (T) is finite ($\mathcal{O}(WT)$). This limit can also be achieved by TDMA and FDMA systems. Thus, using orthogonal-CDMA as the multiple access technique does not give any clear advantages over the conventional TDMA and FDMA. It can however, be shown that in cellular systems, using non-orthogonal signature waveforms does indeed give a capacity improvement over TDMA and FDMA. This is so because unlike TDMA and FDMA, the number of channels in CDMA (non-orthogonal signature waveforms) is unlimited. Hence, the frequency re-use factor does not come into the picture in CDMA systems, thereby giving a capacity improvement over TDMA and FDMA based systems. It is this non-orthogonality in the signature waveforms that causes the Multiple Access Interference (MAI) in CDMA communication systems.

Many CDMA techniques have been proposed in literature (DS-CDMA, FH-CDMA, TH-CDMA, MC-CDMA, etc.). Each of them differs in the way the user signature waveforms are designed. DS-CDMA has been the most popular amongst the CDMA techniques. The condition on orthogonality of the signature waveforms having being dropped, DS-CDMA involves assigning a pseudo-random sequence (PN Code) to each user which acts like the signature waveform of the particular user. The detection is done on the basis of a filter matched to the pseudo-random sequence of the user. We refer to this detector as the *conventional matched filter* detector.

Since the conventional matched filter was designed for orthogonal signature waveforms, it suffers from many drawbacks due to the MAI term which it does not take into account. Multi-user signal processing techniques take into account the MAI term and hence design systems which offer better performance than the conventional matched filter detector.

2 Multi-user Signal Processing

Multi-user signal processing techniques can be broadly classified into two categories:

- *Multi-user Detection*: These are receiver based schemes in which the bulk of the processing is carried out at the receiver end.
- *Multi-user Transmission*: These schemes involve some pre-processing at the transmitter with the aim of keeping the receiver simple. The low computational burden at the receiver makes them good candidates for deployment in the downlink of mobile wireless systems. [2,3,4]

We shall concentrate on *Multi-user Detection* in this article. *Multi-user Transmission* is beyond the scope of this discussion.

3 AWGN Signal Model

Consider a DS-CDMA communication system with K users. Assuming Binary Phase Shift Keying (BPSK) signaling, at the transmitter, the signal for the k^{th} user can be written as

$$x_k(t) = A_k b_k(i) s_k(t - iT_b), \quad iT_b \leq t < (i+1)T_b \tag{1}$$

where,

$$\begin{aligned} s_k(t) &= \frac{1}{\sqrt{N}} \sum_{n=1}^{N} s_{kn} rect(t-(n-1)T_c) && (2) \\ rect(t) &= u(t) - u(t-T_c) && (3) \end{aligned}$$

$u(t)$ is the unit step function, and $b_k(i) \in \{-1, +1\}$. T_b is the bit duration, T_c is the chip duration and $N = \frac{T_b}{T_c}$ is the spreading gain. $\mathbf{s}_k(N \times 1)$ vector is the chip spreading sequence for the k^{th} user.

Define the time-correlation between the signature waveforms of users i and j as

$$\rho_{ij} = \int_0^{T_b} s_i(t) s_j(t) dt \qquad (4)$$

Since more than one user can transmit at the same time, we assume all K users to be simultaneously active. Assuming a synchronous AWGN channel (*i.e.* the data from all users arrives at the receiver at the same instant of time), we can write the received signal at the receiver as follows

$$\begin{aligned} r(t) &= \sum_{k=1}^{K} x_k(t) + n(t) && (5) \\ &= \sum_{k=1}^{K} A_k b_k(i) s_k(t - iT_b) + n(t), \quad iT_b \le t < (i+1)T_b && (6) \end{aligned}$$

where, $n(t)$ is the AWGN noise process with zero mean and variance σ^2. Assuming that the receiver is interested in the data of all users (*e.g.* in the case of uplink communication, this receiver can be the base station), the objective of the receiver is to estimate the vector

$$\mathbf{b}(i) = [b_1(i), \ldots, b_K(i)]^T$$

of transmitted symbols for all time intervals i.

4 Conventional Matched Filter Detector

This is the simplest way to demodulate the received signal: a bank of matched filters, one matched to each users' spreading waveform, is applied to the received signal (Fig. 1). Thus, it demodulates all users independent of each other. Consider the first time interval ($i = 0$) and the j^{th} user,

$$\begin{aligned} r(t) &= \underbrace{A_j b_j s_j(t)} + \underbrace{\sum_{\substack{k=1 \\ k \ne j}}^{K} A_k b_k(i) s_k(t)} + \underbrace{n(t)}, \quad 0 \le t < T_b \\ &= Signal + MAI + Noise \end{aligned} \qquad (7)$$

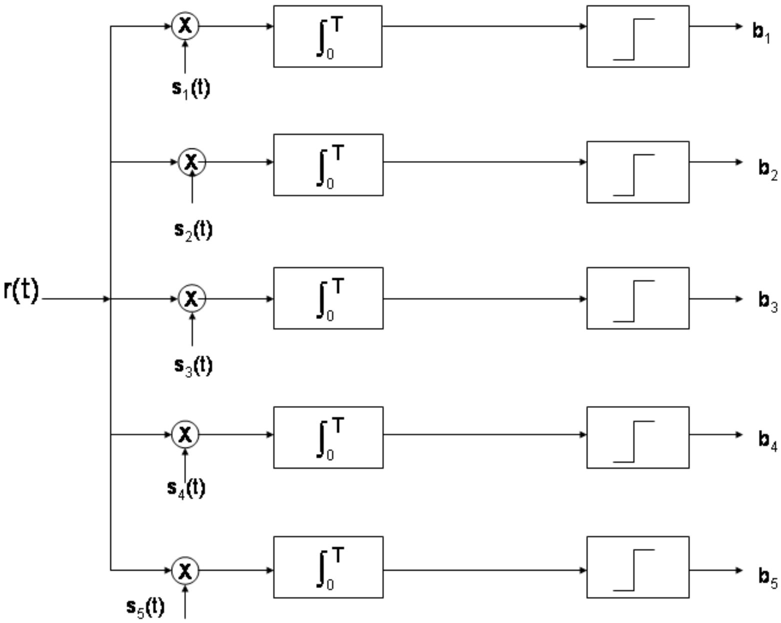

Figure 1: Schematic of the conventional matched filter detector

The statistics of the MAI term are different from the noise term and hence it should be treated differently. Specifically,

- Is an in-band interference unlike noise which is wideband
- Cannot be rejected through a band-pass filter
- Occurs in different forms in other systems also *e.g.* Multi-Carrier interference in OFDM

Since the conventional matched filter detector is designed for the case of orthogonal spreading waveforms, it does not take MAI into account. On applying the same detector to the non-orthogonal case, MAI is treated as a noise term. Thus, correlating the received signal with user j's chip sequence, we get

$$\begin{aligned} y_j &= \int_0^{T_b} r(t) s_j(t) dt \qquad (8) \\ &= A_j b_j + \sum_{\substack{k=1 \\ k \neq j}}^{K} A_k b_k(i) \rho_{kj} + n_j \qquad (9) \end{aligned}$$

Hence,

$$\hat{b}_j = sign(y_j) \qquad (10)$$

Similarly, other users can be demodulated. The conventional matched filter detector

- \+ Is simple to implement
- \+ Does not require knowledge of the channel or the user amplitudes
- – Does not take MAI into account and hence gives non-zero probability of error even with zero noise.
- – Suffers from the *Near-Far Problem*. That is, if the amplitude of one user is higher than others, it will cause a high BER in all other users' demodulated data (due to the MAI term). Thus stringent power control is necessary while using the conventional matched filter detector with non-orthogonal codes.

4.1 Optimality of the Conventional Matched Filter Detector

In the discussion above, we had talked about the MAI term being statistically different from the noise term and hence the need to treat it differently. Note that the MAI term can be approximated as a Gaussian random variable for large number of users (Central Limit Theorem). Does this mean that the matched filter detector will approach the optimum detector for large number of users? The answer is no.

Even if the MAI term was Gaussian, it will still not make the matched filter detector the optimum detector since it demodulates all users independently. In other words, y_j is not a sufficient statistic for b_j, but $[y_1, \ldots, y_K]^T$ is a sufficient statistic for $[b_1, \ldots, b_K]^T$. This is to say that the conventional matched filter will be the optimum detector only if $\rho_{ij} = 0 \quad \forall i, j$.

The optimum detector should take into account the information available in all y_k's to estimate the bit of a particular user. This is known as *Multi-user Detection* and was proposed by Sergio Verdú [1] in the early 1980's. Any multi-user detector will utilise the information available in the MAI term to demodulate the signal and will not treat it like a noise term. This concept is ideologically similar to utilising multi-paths (which were earlier considered as a nuisance) for diversity.

5 Optimum Multi-user Detector

5.1 Individually Optimum Multi-user Detector

Consider the simple 2-user case

$$r(t) = A_1 b_1 s_1(t) + A_2 b_2 s_2(t) + n(t), \quad 0 \le t < T_b \tag{11}$$

The optimum estimate of b_1 will minimise the probability of error and is obtained by choosing $\hat{b}_1 \in \{-1, +1\}$ such that the aposteriori probability $P(b_1 = \hat{b}_1 | r(t), 0 \leq t < T_b)$ is maximised. Similarly for user 2, *i.e.* we need to choose $\hat{b}_2$ such that $P(b_2 = \hat{b}_2 | r(t), 0 \leq t < T_b)$ is maximised.

This detector can be termed as an *individually optimum multi-user detector*

5.2 Jointly Optimum Multi-user Detector

The individually optimum detector is not optimum since $\hat{b}_1$ and $\hat{b}_2$ are not independent conditioned on the received signal $r(t)$. Thus, we need to maximise the joint aposteriori probability

$$P(b_1 = \hat{b}_1, b_2 = \hat{b}_2 | r(t), 0 \leq t < T_b)$$

This detector can be termed as a *jointly optimum* detector. This is also the globally optimum multi-user detector. Now consider the general K-user case

$$r(t) = \sum_{k=1}^{K} A_k b_k s_k(t) + n(t), \quad 0 \leq t < T_b \tag{12}$$

For equal apriori probabilities of all $\mathbf{b} = [b_1, \ldots, b_K]^T$, maximising $P(\mathbf{b} = \hat{\mathbf{b}} | r(t), 0 \leq t < T_b)$, is equivalent to maximising

$$f(r(t), 0 \leq t < T | \mathbf{b} = \hat{\mathbf{b}}) = exp\left(-\frac{1}{2\sigma^2}\int_0^{T_b}\left[r(t) - \sum_{k=1}^{K} A_k \hat{b}_k s_k(t)\right]^2 dt\right) \tag{13}$$

That is, choosing $\hat{\mathbf{b}}$ such that $\sum_{k=1}^{K} A_k \hat{b}_k s_k(t)$ is closest to the received signal in the mean square sense. Note that

- + The jointly optimum multi-user detector minimises the probability of error
- + It gives zero probability of error in the absence of noise
- + It does not suffer from the near-far problem
- – It needs knowledge about the channel characteristics at the receiver
- – Finding the optimum solution is an exponentially complex task $\mathcal{O}(2^K)$ (K - no. of users), even for the AWGN channel and no polynomial time algorithm is known. Moreover, the problem has been shown to be NP-complete [5].

Therefore, there is a need to look for sub-optimum approaches to multi-user detection.

6 Sub-Optimum Multi-user Detectors

6.1 Linear Multi-user Detectors

These class of algorithms involve applying a linear transformation to the matched filter (single user detector) outputs. The output of the matched filter can be written in matrix form as

$$\mathbf{y}_{MF} = \mathbf{RAb} + \mathbf{n} \tag{14}$$

where, $\mathbf{R} = \mathbf{S}^T\mathbf{S}$ is the correlation matrix of the user chip sequences (pseudo-random sequences).

6.1.1 Decorrelating Detector

The decorrelating receiver applies the inverse of the correlation matrix to the output of the matched filter in order to decouple the data. *i.e.*

$$\hat{\mathbf{b}} = sign(\mathbf{R}^{-1}\mathbf{y}_{MF}) \tag{15}$$

This detector,

- + Completely eliminates the MAI, hence is near-far resistant
- + Does not require estimates of the channel parameters
- – Enhances the noise

6.1.2 MMSE Detector

The MMSE detector implements a linear mapping $\mathbf{L}$ which minimises the mean squared error $E(|\mathbf{b} - \mathbf{L}\mathbf{y}_{MF}|^2)$. The detection scheme can be written as

$$\hat{\mathbf{b}} = sign(\mathbf{L}\mathbf{y}_{MF}) \tag{16}$$

This detector,

- + Performs better than the decorrelating detector since it takes noise into account
- – Requires an estimate of the channel at the receiver

6.2 Non-linear Multi-user Detectors

This class of detectors apply non-linear processing to the matched filter outputs.

6.2.1 Successive Interference Cancellation (SIC)

This involves serially canceling the interfering users from the outputs of the matched filters in order of decreasing power. Thus

$$\hat{b}_j = sign\left(y_j - \sum_{k=j+1}^{K} A_k \rho_{kj} \hat{b}_k\right) \tag{17}$$

where the users have been ranked 1 to K according to increasing received signal strength. The SIC detector

- + Is easy to implement
- – Suffers from the drawback of error propagation
- – Requires channel estimates to be available at the receiver

6.2.2 Parallel Interference Cancellation (PIC)

Like the SIC detector, this detector also involves subtracting the interference of other users. Unlike the serial subtraction in SIC, the PIC detector cancels the estimates of the MAI from the outputs of the matched filters in a parallel manner. It follows an iterative process. Thus, for $j = 1, \ldots K$

$$\hat{b}_j^{(m+1)} = sign\left(y_j - \sum_{k \neq j}^{K} A_k \rho_{kj} \hat{b}_k^{(m)}\right) \tag{18}$$

Issues with the PIC detector are

- – The bit decisions need not converge
- – The performance depends heavily on the initial bit estimates
- – Requires channel estimates at the receiver

6.3 Adaptive Multi-user Detectors

This covers a large number of algorithms which are adaptive in nature and can adapt to unknown and time-varying channel conditions through training or other blind mechanisms. Their adaptive nature makes them more suitable for deployment in practical systems than the algorithms in the previous sections. Examples of this class of algorithms include training based RLS, training based LMS and gradient descent based MMSE optimisation.

We will look at two multi-user detection algorithms in Chapter **??** and present a modification to them which not only improves the BER performance, but also reduces the computational burden.

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