# Efficient Minimum Probability of Error Demodulation for DS-CDMA Systems

Mohit Garg, Umesh D. Nimbhorkar, U. B. Desai, S. N. Merchant
Dept. of Electrical Engineering
Indian Institute of Technology
Mumbai, India
Email: {mohitgarg, umeshdn}@iitb.ac.in, {ubdesai, merchant}@ee.iitb.ac.in

***Abstract*— Demodulators based on the Minimum Mean Squared Error (MMSE) criterion are optimal for 'ideal' AWGN channels. However, In the case of wireless multipath channels, wherein ISI is inherent, MMSE based algorithms do not offer the optimal framework for demodulation. Minimum Probability of Symbol Error (MPOSE) based detectors have been shown to perform significantly better than MMSE based approaches in these scenarios under a variety of modulation and multiplexing schemes. High computational complexity, however, prohibits the use of these algorithms in practical communication systems. The major challenge now is to reduce the computational burden of the MPOSE algorithms.**

**This paper, as a step in this direction, discusses two already published MPOSE algorithms and proposes a modification to them whereby we not only achieve significant gains in computational complexity but also improve the BER performance. The system is also found to be robust to error propagation due to wrong estimation of the ISI term.**

## I. INTRODUCTION

Wireless technologies are being developed at an exponential pace with newer applications being released everyday. This fast growth in popularity and customer base has led to a lot of techniques from the wired world being deployed directly to the wireless scenarios without a thorough study on the optimality of the algorithms. MMSE based demodulation schemes are one such example being used extensively at the physical layer in wireless systems.

The optimality of MMSE based algorithms is well-proven for Additive White Gaussian Noise (AWGN) channels. Wired channels being 'very close' to 'ideal' AWGN channels, most conventional detectors used the MMSE criterion for demodulation and detection of digital symbols. The advent of wireless technologies saw these being directly adapted to the wireless scenarios where they are being used till date. Wireless channels are incomparably more hostile and different from wired channels and hence the performance of MMSE based approaches is severely degraded and much below optimal for these systems. Thus there is a necessity to develop optimal algorithms for demodulation in multipath, ISI inducing wireless channels.

Minimum Probability of Error (MPOE) turns out to be the most natural choice for the optimality criterion for digital communication systems. A lot of work has been done in this direction [1] and MPOE based demodulators have been proposed for a wide variety of multiplexing schemes *viz.* DS-CDMA [2], OFDM-SDMA [3], [4], and MC-CDMA [5], [6].

All the above algorithms are adaptive multiuser detection [7] algorithms and perform significantly better than the adaptive MMSE based multiuser algorithms as reported in the respective papers. However, one common issue faced by all the MPOE based algorithms is the high computational burden at the receiver for searching over the possible state space for every received symbol. This severely restricts the practical deployment of these algorithms.

This paper focuses on multiuser detection algorithms for DS-CDMA systems [8]–[10] employing single transmit and multiple receive antennas [11], [12]. We take up the algorithms proposed in [2] and present a modification to them which not only reduces the computational burden but also improves the BER performance. The modification is based on an observation on the decision statistic which is slightly modified to allow for pre-computation of weights. The new scheme also reduces the number of flops per iteration during the weight updation phase and the convergence is also smoother as shown in Fig. 1. Let

$$\eta = \frac{T_{chan}}{T_{bit}} \tag{1}$$

where, $T_{chan}$ is the expected channel variation timescale and $T_{bit}$ is the symbol duration. Thus, the expected overall gain in computation is greater than $\eta$.

In the next section, we introduce the algorithms proposed in [2]. Section III formulates the problem, section IV is a detailed discussion on the algorithms. We present our modification and simulation results in section V. Conclusions and intended future work directions are discussed in section VI.

## II. MINIMUM PROBABILITY OF ERROR DEMODULATION

Two adaptive space-time multiuser detectors based on the criterion of *Minimum Probability of Error* have been proposed in [2]. Multi-path, asynchronous DS-CDMA channel with single transmit and multiple receive antennas is assumed in the problem formulation.

The first algorithm, Minimum Joint Probability Of Error (MJPOE), minimises the joint probability of error for all users. This algorithm has a computational complexity which is exponential in the number of users and the weights need to be adapted for each incoming bit vector. The modification that we propose not only reduces the BER further, but also makes the computation of weights a one-time affair *i.e.* we can pre-compute the weights and use them till the system specifications change. This reduces the computational complexity of demodulation in ISI dominant asynchronous channels to levels comparable to those in synchronous environments. Thus weight computation is now done on the time scale of the variations in the channel rather than the bit duration.

The second algorithm, Minimum Conditional Probability Of Error (MCPOE), is a compromise on the BER w.r.t MJPOE but is the computational complexity is linear in the number of users. Here too, the proposed modification shows significant reduction in BER. Since the computation of weights in this case is conditioned on the transmitted vector during the training phase, pre-computation of weights is not possible in this algorithm.

We first discuss the original algorithms and then present the proposed modification in the next few sections.

## III. Signal Model

Consider a system with $K$ users, $M$ multi-paths in each user's channel, and $P$ receive antennas. Let $g_{km}$ and $\tau_{km}$ respectively denote the complex gain and delay of the $m^{\text{th}}$ multi-path of the $k^{\text{th}}$ user's signal, and $[a_{km,1},\ldots,a_{km,P}]^T$ is the antenna array response vector corresponding to the $m^{\text{th}}$ path of the $k^{\text{th}}$ user's signal.

Let the transmitted waveform for the $k^{\text{th}}$ user be represented by

$$x_k(t) = A_k b_k(i) c_k(t - iT_{bit}), \qquad iT_{bit} \le t < (i+1)T_{bit} \quad (2)$$

where, $A_k$ is the transmit amplitude of the $k^{\text{th}}$ user, $b_k(i) = \underline{+}1$ denotes the transmitted bit for the user at time instant $i$, and $c_k(.)$ is the chip waveform for the particular user. Let the processing gain $T_{bit}/T_c$ be denoted by $N$. Also define, $n_{km} = \tau_{km}/T_c$. The assumption made is that all multi-path delays are multiples of the chip period and hence $n_{km}$ will be an integer. For the sake of simplicity, it is assumed that the multi-path delay of any user does not exceed the symbol period so that the contribution from $(i-2)^{\text{th}}$ and all preceeding signaling intervals are zero.

Each antenna will now sample the received signal $N$ times during each bit duration, thereby giving a $N \times P$ received vector. From the analysis in [2], this received signal vector for the $p^{\text{th}}$ antenna can be written as

$$\mathbf{r}_p(i) = \mathbf{S}_{pL}\mathbf{A}\mathbf{b}(i-1) + \mathbf{S}_{pR}\mathbf{A}\mathbf{b}(i) + \sigma\mathbf{n}_p \quad (3)$$

where $\mathbf{n}_p \ \ (P \times 1)$ is a white Gaussian noise vector and $\sigma^2$ is the variance of the ambient noise at each antenna element. Also,

$$\begin{aligned} \mathbf{S}_{pL} &= [\alpha_{1p,L}\ldots\alpha_{Kp,L}] && (N \times K) \\ \mathbf{S}_{pR} &= [\alpha_{1p,R}\ldots\alpha_{Kp,R}] && (N \times K) \\ \mathbf{A} &= diag(A_1,\ldots,A_K) && (K \times K) \end{aligned} \quad (4)$$

and

$$\begin{aligned} \alpha_{kp,L} &= \sum_{m=1}^{M} a_{km,p} g_{km} \mathbf{s}_{kL}^{n_{km}} \\ \alpha_{kp,R} &= \sum_{m=1}^{M} a_{km,p} g_{km} \mathbf{s}_{kR}^{n_{km}} \end{aligned} \quad (5)$$

$\mathbf{s}_{kL}^n$ and $\mathbf{s}_{kR}^n$ are the shifted chip sequences of the $k^{\text{th}}$ user. This represents the asynchronous nature of the channel.

$$\mathbf{s}_{kL}^n = [\underbrace{s_{kN-n+1},\ldots,s_{kN}}_{n}, \underbrace{0,\ldots,0}_{N-n}]^T \quad (6)$$

$$\mathbf{s}_{kR}^n = [\underbrace{0,\ldots,0}_{n}, \underbrace{s_{k1},\ldots,s_{kN-n}}_{N-n}]^T \quad (7)$$

Here $s_{kl}$ denotes the $l^{\text{th}}$ element ($1 \le l \le N$) of the chip sequence of the $k^{\text{th}}$ user.

## IV. MPOE Algorithms

### A. Receiver Structure

Now that the expression for the received signal at each antenna of the receiver has been evaluated (3), we turn our attention to the demodulation process. Define the augmented $(NP \times 1)$ received vector by $\mathbf{r} = [\mathbf{r}_1^T,\ldots,\mathbf{r}_P^T]^T$. The receiver structure consists of a linear filter which operates on this augmented vector. Let the weights for the $k^{\text{th}}$ user be denoted by $\mathbf{w}_k = [\mathbf{w}_{k1}^T,\ldots,\mathbf{w}_{kP}^T]^T \quad (NP \times 1)$ .

The output of the filter $\mathbf{w}_k$ in the $i^{\text{th}}$ bit interval is

$$\begin{aligned} y_k(i) &= \mathbf{w}_k^H \mathbf{r}(i) \\ &= \sum_{p=1}^{P} \mathbf{w}_{kp}^H \mathbf{r}_p(i) \\ &= \sum_{p=1}^{P} \mathbf{w}_{kp}^H \mathbf{S}_{pL}\mathbf{A}\mathbf{b}(i-1) + \sum_{p=1}^{P} \mathbf{w}_{kp}^H \mathbf{S}_{pR}\mathbf{A}\mathbf{b}(i) \\ &\quad + \sigma \mathbf{w}_k^H \mathbf{n} \end{aligned} \quad (8)$$

where $(.)^H$ denotes the Hermitian operator and $\mathbf{n} = [\mathbf{n}_1^T,\ldots,\mathbf{n}_P^T]^T$. The bit decision for the $k^{\text{th}}$ user is given by

$$\hat{b}_k = sgn[\Re(y_k)] \quad (9)$$

where $sgn[.]$ denotes the signum function and $\Re(.)$ denotes the real part. Since the bit vector $\mathbf{b}(i-1)$ has been detected in the $(i-1)^{\text{th}}$ interval, we can replace $\mathbf{b}(i-1)$ by its estimate $\hat{\mathbf{b}}(i-1)$. Thus we get

$$y_k = \zeta_k + \sum_{p=1}^{P} \mathbf{w}_{kp}^H \mathbf{S}_{pR}\mathbf{A}\mathbf{b} + \sigma \mathbf{w}_k^H \mathbf{n} \quad (10)$$

where we have dropped the index $i$ for notational convenience. Also,

$$\zeta_k = \sum_{p=1}^{P} \mathbf{w}_{kp}^H \mathbf{S}_{pL} \mathbf{A} \hat{\mathbf{b}}(i-1) \tag{11}$$

denotes the estimated ISI term.

### B. MJPOE

Let $\mathbf{y} = [y_1, \ldots, y_k]^T \quad (K \times 1)$. Since the receiver decision depends on $\Re(\mathbf{y})$, the probability of error will depend on the probability distribution of $\mathbf{y}$. Following the analysis in [2], one can derive the joint probability of error of all users to be

$$P_E = 1 - \frac{1}{2^K} \sum_{\forall \mathbf{b}} \prod_{k=1}^{K} Q\left(-\frac{b_k \mu_k}{\sigma_k}\right) \tag{12}$$

Here the sum is over all possible $K$ dimensional transmitted vectors ($2^K$ in number, thus giving the 'exponential in number of users' complexity). $\mu_k$ and $\sigma_k^2$ are respectively the conditional (conditioned on $\mathbf{b}$) expectation and variance of $y_k$ given by

$$\begin{aligned}\mu_k &= E[\Re(y_k)] \\ &= \Re\left(\zeta_k + \sum_{p=1}^{P} \mathbf{w}_{kp}^H \mathbf{S}_{pR} \mathbf{A} \mathbf{b}\right)\end{aligned} \tag{13}$$

and

$$\sigma_k^2 = \sigma^2 \|\mathbf{w}_k\|^2 \tag{14}$$

In order to find the weights that minimise the $P_E$ above, a gradient descent technique coupled with Gram-Schmidt orthogonalisation is used in [2]. The expression for the gradient can be derived to be

$$\frac{\partial P_E}{\partial \mathbf{w}_k} = -\frac{1}{\sqrt{2\pi.2^K}} \sum_{\forall \mathbf{b}} \left( b_k e^{-\mu_k^2/2\sigma_k^2} \phi_k \prod_{\substack{i=1 \\ i \neq k}}^{K} Q\left(-\frac{b_i \mu_i}{\sigma_i}\right)\right) \tag{15}$$

where $\phi_k = [\phi_{k1}^T, \ldots, \phi_{kP}^T]^T$ is a $NP \times 1$ vector such that for $1 \leq p \leq P$,

$$\phi_{kp} = \frac{1}{\sigma_k}\left(\Re(\mathbf{S}_{pL} \mathbf{A} \hat{\mathbf{b}}(i-1)) + \Re(\mathbf{S}_{pR} \mathbf{A} \mathbf{b}) - \frac{\mu_k \mathbf{w}_k}{\mathbf{w}_k^H \mathbf{w}_k}\right) \tag{16}$$

The converged weights are then used to demodulate the incoming signal. This is the Minimum Joint Probability of Error (MJPOE) algorithm.

### C. MCPOE

The expression for the joint probability (12) is a sum of $2^K$ terms with each term corresponding to one bit vector of length $K$. Hence, the MJPOE algorithm is exponentially complex in the number of users. In order to reduce this complexity at the cost of a marginal increase in the expected BER, the authors in [2] present the Minimum Conditional Probability of Error (MCPOE) algorithm.

This algorithm minimises the conditional probability of error $P_{k|\mathbf{b}}$, conditioned on the transmitted bit vector $\mathbf{b}$ for each user individually. Here too a gradient descent approach is adopted for adaptively computing the weights which minimise the conditional probability of error. This reduces the computational complexity to linear in the number of users.

The mathematical details are identical to the MJPOE algorithm except for the gradient descent phase wherein $P_{k|\mathbf{b}}$ is used instead of $P_E$. Reproducing the expressions from [2]

$$P_{k|\mathbf{b}} = \frac{1}{2} Q\left(\frac{\mu_{k|1}}{\sigma_k}\right) + \frac{1}{2} Q\left(\frac{-\mu_{k|-1}}{\sigma_k}\right) \tag{17}$$

where $\mu_{k|1}$ and $\mu_{k|-1}$ are the conditional expectations of $y_k$. Also,

$$\begin{aligned}\frac{\partial P_{k|\mathbf{b}}}{\partial \mathbf{w}_k} = &-\frac{1}{\sqrt{2\pi}} exp\left(\frac{-\mu_{k|1}^2}{\sigma_k^2}\right) \frac{\partial}{\partial \mathbf{w}_k}\left(\frac{\mu_{k|1}}{\sigma_k}\right) \\ &+ \frac{1}{\sqrt{2\pi}} exp\left(\frac{-\mu_{k|-1}^2}{\sigma_k^2}\right) \frac{\partial}{\partial \mathbf{w}_k}\left(\frac{\mu_{k|-1}}{\sigma_k}\right)\end{aligned} \tag{18}$$

The partial derivatives can be computed as follows

$$\frac{\partial}{\partial \mathbf{w}_k}\left(\frac{\mu_{k|1}}{\sigma_k}\right) = \frac{\|\mathbf{w}_k\|^2 \mathbf{u}_k - \mu_{k|1} \mathbf{w}_k}{\sigma \|\mathbf{w}_k\|^{3/2}} \tag{19}$$

$$\frac{\partial}{\partial \mathbf{w}_k}\left(\frac{\mu_{k|-1}}{\sigma_k}\right) = \frac{\|\mathbf{w}_k\|^2 \mathbf{v}_k - \mu_{k|-1} \mathbf{w}_k}{\sigma \|\mathbf{w}_k\|^{3/2}} \tag{20}$$

where $\mathbf{u}_k = [\mathbf{u}_{k1}^T, \ldots, \mathbf{u}_{kP}^T]^T$ and $\mathbf{v}_k = [\mathbf{v}_{k1}^T, \ldots, \mathbf{v}_{kP}^T]^T$ are $NP \times 1$ vectors and can be shown to be

$$\begin{aligned}\mathbf{u}_{kp} = &\Re\left(\mathbf{S}_{pL} \mathbf{A} \hat{\mathbf{b}}(i-1) + \sum_{\substack{j=1 \\ j \neq k}}^{K} A_j b_j \sum_{m=1}^{M} a_{jm,p} g_{jm} \mathbf{s}_{jR}^{n_{jm}}\right) \\ &+ \Re\left(A_k \sum_{m=1}^{M} a_{km,p} g_{km} \mathbf{s}_{kR}^{n_{km}}\right)\end{aligned} \tag{21}$$

$$\begin{aligned}\mathbf{v}_{kp} = &\Re\left(\mathbf{S}_{pL} \mathbf{A} \hat{\mathbf{b}}(i-1) + \sum_{\substack{j=1 \\ j \neq k}}^{K} A_j b_j \sum_{m=1}^{M} a_{jm,p} g_{jm} \mathbf{s}_{jR}^{n_{jm}}\right) \\ &- \Re\left(A_k \sum_{m=1}^{M} a_{km,p} g_{km} \mathbf{s}_{kR}^{n_{km}}\right)\end{aligned} \tag{22}$$

The MCPOE algorithm as such requires a training sequence for computing the optimal weights. A blind version of the MCPOE algorithm is also proposed in the same paper wherein estimates from a conventional matched filter detector are used as the target sequence. The convergence curves for the blind and training based MCPOE lie close to each other as reported in the paper.

We now proceed onto the proposed modification to the algorithms which enables pre-computation of weights in the MJPOE algorithm and improves BER performance.

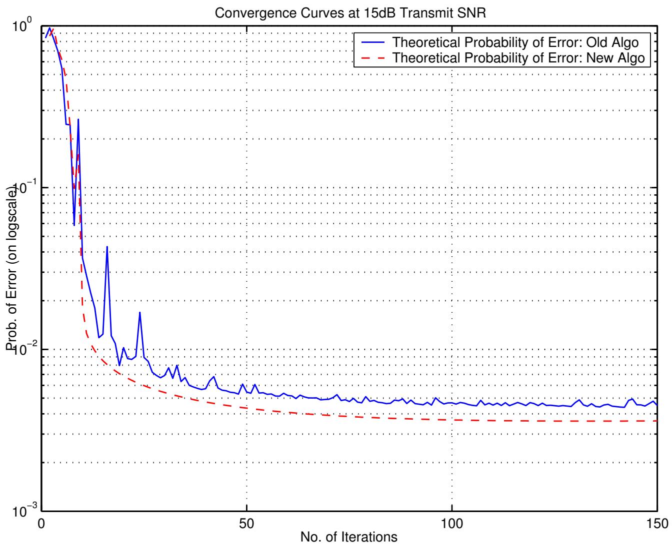


Fig. 1. Convergence curves for both the schemes at 15dB transmit SNR

## V. Modification to the MPOE Algorithms

We propose a modification in the decision statistic $y_k$ in (8) *i.e.* define

$$z_k = y_k - \zeta_k \tag{23}$$

where $\zeta_k$ is the estimated ISI component in $y_k$ expressed in (11). The decision rule after this ISI cancellation becomes

$$\hat{b}_k = sgn[\Re(z_k)] \tag{24}$$

The intuition behind this is that since the original decision rule in (9) uses zero as the threshold, the expectation of the original decision statistic $y_k$ should also be zero for optimal demodulation [13]. This is not the case due to the additional estimated ISI term $\zeta_k$ in the expression for $y_k$. Since $\zeta_k$ is a known term which varies at each bit interval we should either make the threshold adaptive to $\zeta_k$ or subtract out $\zeta_k$ before making the decision. We follow the latter approach due to its simplicity.

Thus, $z_k$ is the new decision statistic with $E(z_k) = 0$ and no ISI term involved in it. This modification helps us in two ways

- Firstly, it is expected to improve the BER performance since we are now using the optimal decision statistic - decision rule combination.
- Secondly, since the new decision statistic $z_k$ does not involve any ISI term, and the joint probability of error expression does not involve the actual value of the decision statistic (only the statistics of the decision statistic are needed), we should be able to pre-compute the weights and keep them as long as the channel remains invariant. This is precisely what is seen on working out the details *i.e.* the new weight update equation does not contain any term which varies from one bit interval to another.

In (23) we are directly subtracting the estimated ISI term from the decision statistic which may lead to error propagation. Simulation results presented later show that this is not the case and that the algorithm is robust to errors in ISI estimation.

Proceeding along similar lines as in [2], one can show that

$$\tilde{P}_E = 1 - \frac{1}{2^K} \sum_{\forall \mathbf{b}} \prod_{k=1}^{K} Q\left(-\frac{b_k \nu_k}{\sigma_k}\right) \tag{25}$$

where the tilde is used to distinguish the new probability of error from the earlier one in (12). Also $\nu_k$ and $\sigma_k^2$ are respectively the conditional (conditioned on $\mathbf{b}$) expectation and variance of $z_k$ given by the following equations. Note that the variance remains unaltered since we are only subtracting a deterministic quantity $\zeta_k$ from $y_k$ in (23).

$$\begin{aligned} \nu_k &= E[\Re(z_k)] \\ &= \Re\left(\sum_{p=1}^{P} \tilde{\mathbf{w}}_{kp}^H \mathbf{S}_{pR} \mathbf{A} \mathbf{b}\right) \end{aligned} \tag{26}$$

and

$$\sigma_k^2 = \sigma^2 \|\tilde{\mathbf{w}}_k\|^2 \tag{27}$$

The gradient can now be written as

$$\frac{\partial \tilde{P}_E}{\partial \tilde{\mathbf{w}}_k} = -\frac{1}{\sqrt{2\pi.2^K}} \sum_{\forall \mathbf{b}} \left( b_k e^{-\nu_k^2/2\sigma_k^2} \tilde{\phi}_k \prod_{\substack{i=1 \\ i \neq k}}^{K} Q\left(-\frac{b_i \nu_i}{\sigma_i}\right)\right) \tag{28}$$

where $\tilde{\phi}_k = [\tilde{\phi}_{k1}^T, \ldots, \tilde{\phi}_{kP}^T]^T$ is a $NP \times 1$ vector such that for $1 \leq p \leq P$,

$$\tilde{\phi}_{kp} = \frac{1}{\sigma_k}\left(\Re(\mathbf{S}_{pR}\mathbf{A}\mathbf{b}) - \frac{\nu_k \tilde{\mathbf{w}}_k}{\tilde{\mathbf{w}}_k^H \tilde{\mathbf{w}}_k}\right) \tag{29}$$

Note that none of (25), (28) and (29) involve any term which depends on the actual transmitted symbol. This is so because we are summing over all possible transmitted symbols in calculating the joint probability of error. Also, none of these equations contain any term which varies between bit intervals. This allows pre-computation of weights thereby restricting weight updation to channel variation timescales rather than bit timescales as in the previous algorithm.

The same modification when applied to both versions of the MCPOE algorithm gives a significant improvement in BER performance. Pre-computation of weights is not possible in the MCPOE algorithm since the expression for the probability of error is conditioned on the transmitted vector during the training phase.

### A. Simulation Results

Extensive simulations were carried out using the above model. An asynchronous channel with $K = 4$ users, $M = 3$ multi-paths per user, and a spreading gain of $N = 15$ was simulated. Equiprobable transmit bits $b_k = \pm 1$ were assumed.

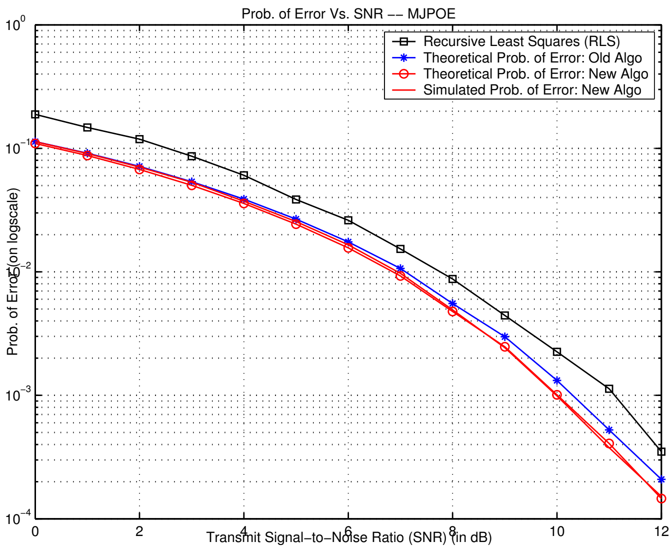


Fig. 2. Per-User BER performance comparison for MJPOE algorithm averaged over 15 independent channels

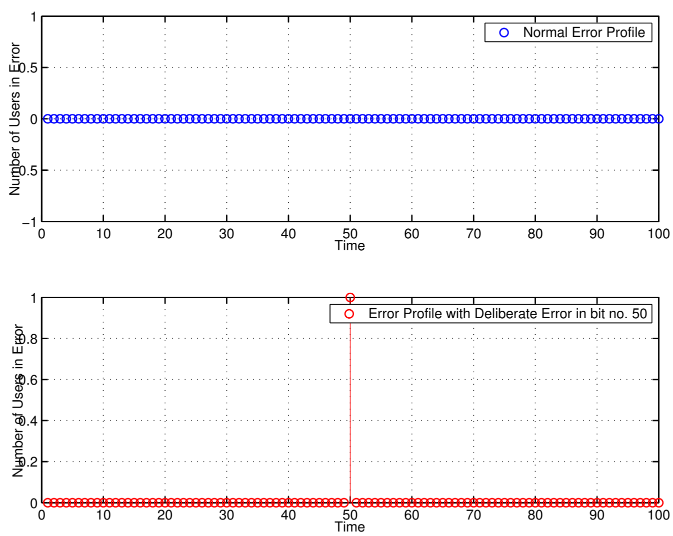


Fig. 3. Error Propagation Results in modified MJPOE due to deliberately induced error in ISI estimation. SNR = $7dB$.

The receiver was assumed to consist of a linear array of $P = 3$ antennas with half wavelength spacing and response given by

$$a_{km,p} = e^{j(p-1)\pi sin(\theta_{km})} \tag{30}$$

where, $j = \sqrt{(-1)}$, $\theta_{km}$ is the direction of arrival (DOA) of the $k^{\text{th}}$ user along the $m^{\text{th}}$ path w.r.t. the antenna array. Multi-path gains ($U(0,1)e^{jU(-\pi/2,\pi/2)}$), DOA ($U(-\pi/2,\pi/2)$) and propagation delays ($\tau_k \leq T_{bit} \quad \forall k$) were randomly generated for all users. $10,000$ bits were used to generate plots of the measured quantities. Transmit SNR was used as the SNR measure in all the plots [14].

The results reported were consistently observed across a large number of independent scenarios.

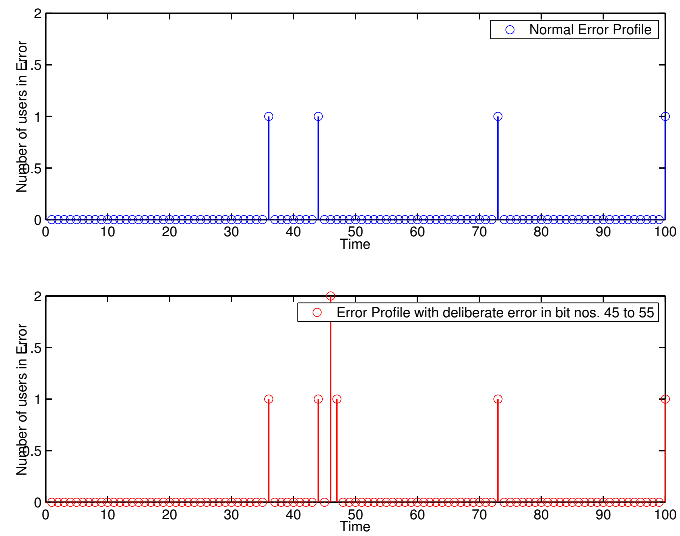


Fig. 4. Error recovery results in modified MJPOE due to consecutive induced errors in ISI estimation. SNR=$7dB$.

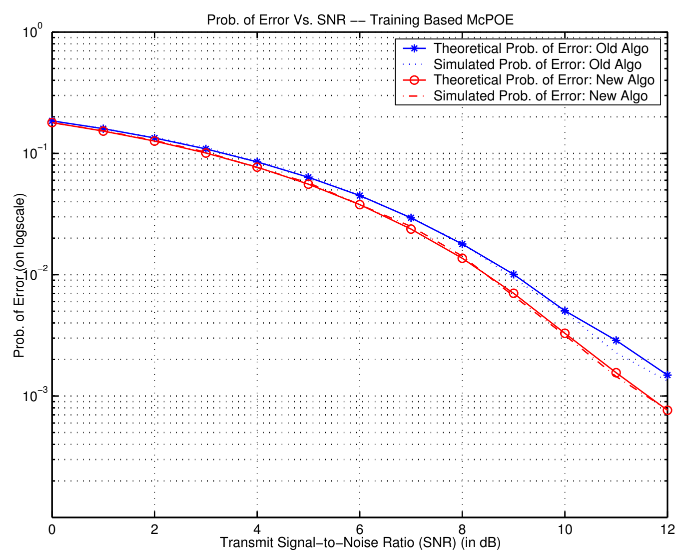


Fig. 5. Training based MCPOE algorithm. Averaged over 10 independent channels. (per-user BER)

Convergence results from the MJPOE algorithm are presented in Fig. 1. It is clear that the modified algorithm gives a smoother convergence curve than the original algorithm. This is expected because the ISI term varies at each bit interval and hence the optimal weights will slightly vary with each bit in the original algorithm.

The BER results for the MJPOE algorithm are shown in Fig. 2. Both the schemes outperform the RLS adaptive algorithm in terms of BER. Also, the modified scheme performs better than the original scheme at higher SNRs whereas the performance is almost identical at lower SNRs. This is due the fact that at lower SNRs the noise term dominates the error term and hence the sub-optimal decision statistic due to the ISI term in the original scheme is not the dominant cause of

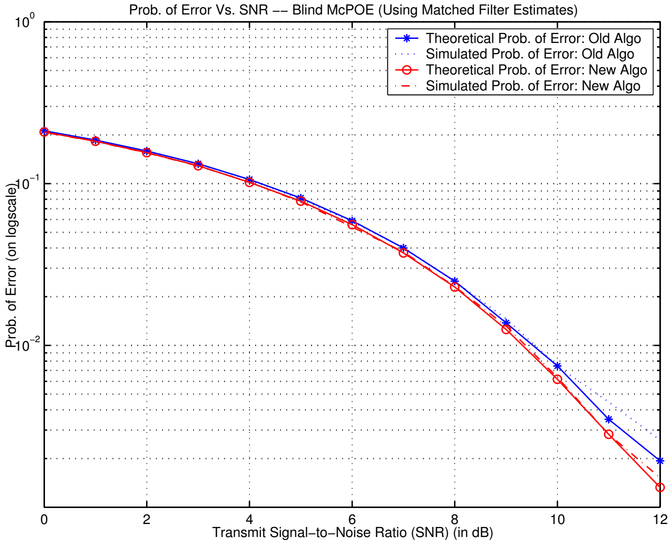


Fig. 6. Blind MCPOE algorithm using estimates from a matched filter. Averaged over 10 independent channels. (per-user BER)

error. At higher SNRs, however, the noise term is superseded by the ISI term and hence we see a significant improvement the BER performance of the new algorithm *vis-a-vis* the earlier algorithm. This plot shows the per user probability of error.

The modified algorithm involves direct subtraction of the estimate of the ISI term from the original decision statistic as in (23). This may lead to error propagation in case the previous decoded bit vector is in error. We also simulated this aspect of the algorithm. Our simulations showed that the errors were not clustered together showing absence of error propagation and that recovery from error was fast. Fig. 3 reports the results of a simulation wherein we deliberately induced an error in a particular bit. It is clear that the error does not propagate. Fig. 4 shows a case when we subtracted the incorrect estimate of the ISI term using wrong bits for all users in evaluating $\zeta_k$ as in (23) for a number of bits in succession. Thus for the environment we considered, the algorithm shows fast error recovery and robustness to wrong ISI estimation.

Simulating both variants of the MCPOE algorithm results in Figs. 5 and 6. Here too the modified scheme performs better than the original algorithm. Again we see a larger difference at higher SNRs as compared to lower SNRs due to the dominant effect of noise at low SNRs.

Thus, the suggested modification not only reduces computation in MJPOE but also improves the performance of both the algorithms.

## VI. Conclusion

It can be stated without apprehension that in digital communications, the emphasis should be on Minimum Probability of Symbol Error (MPOSE) based approaches rather than energy based error analysis. The proposed scheme reduces the weight updation process in ISI dominant asynchronous channels to channel variation timescales, hence requiring only a vector dot product at the receiver for demodulation. Better BER performance and smoother convergence profiles are also an added advantage with the new algorithm.

Future work may entail exploring convergence techniques better than the gradient descent algorithm for further reducing the computational burden. Indeed, a MPOSE algorithm with tractable computation will be a strong candidate for implementation in next generation systems.

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