Efficient Minimum Probability Of Error Demodulation for DS-CDMA Systems

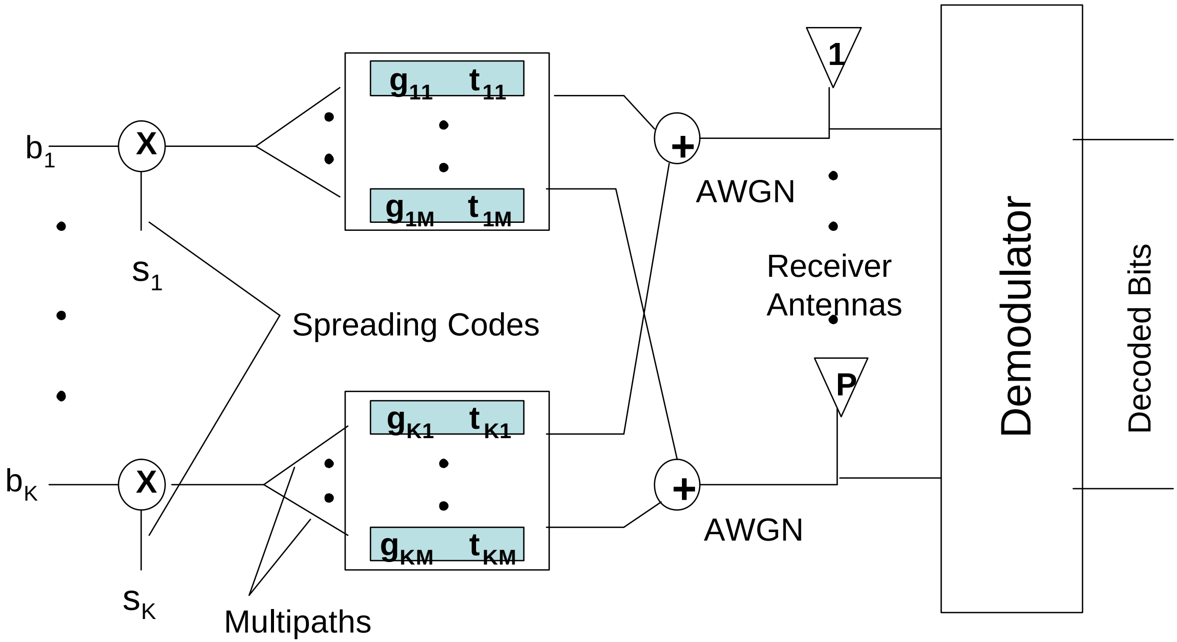
Mohit Garg, Umesh D. Nimbhorkar, U. B. Desai, S. N. Merchant

{mohitgarg,umeshdn}@iitb.ac.in, {ubdesai,merchant}@ee.iitb.ac.in

SPANN Lab
Department of Electrical Engineering
IIT-Bombay

Multiuser Detection

- Maximum Likelihood Multiuser Detector formulated by Verdu (1986) - significant capacity and performance improvement over the conventional detector
  - Computational complexity of ML-MUD - $O(2^K)$, for K users
- Several sub-optimal linear multiuser detection algorithms proposed in literature (MMSE, MOE, etc.): Madhow, Poor, Wang *et al.*
- Trade-off between computational complexity and performance


b1
bK
X
X
s1
sK
g11 t11
g1M t1M
gK1 tK1
gKM tKM
Spreading Codes
Multipaths
+
+
AWGN
AWGN
1
P
Receiver
Antennas
Demodulator
Decoded Bits

Multiuser Detection: The Proposed Approach

In earlier work, we proposed two schemes :

- ***MJPOE*** : Minimum Joint Probability of Error
- ***MCPOE*** : Minimum Conditional Probability of Error
  in a space-time framework
- *The basic premise being that probability of error* (BER or symbol error rate) *is a better error measure than mean square error when dealing symbols*

- **In this work, we propose a modification to the decision statistic which not only reduces the computation but also improves the BER performance.**

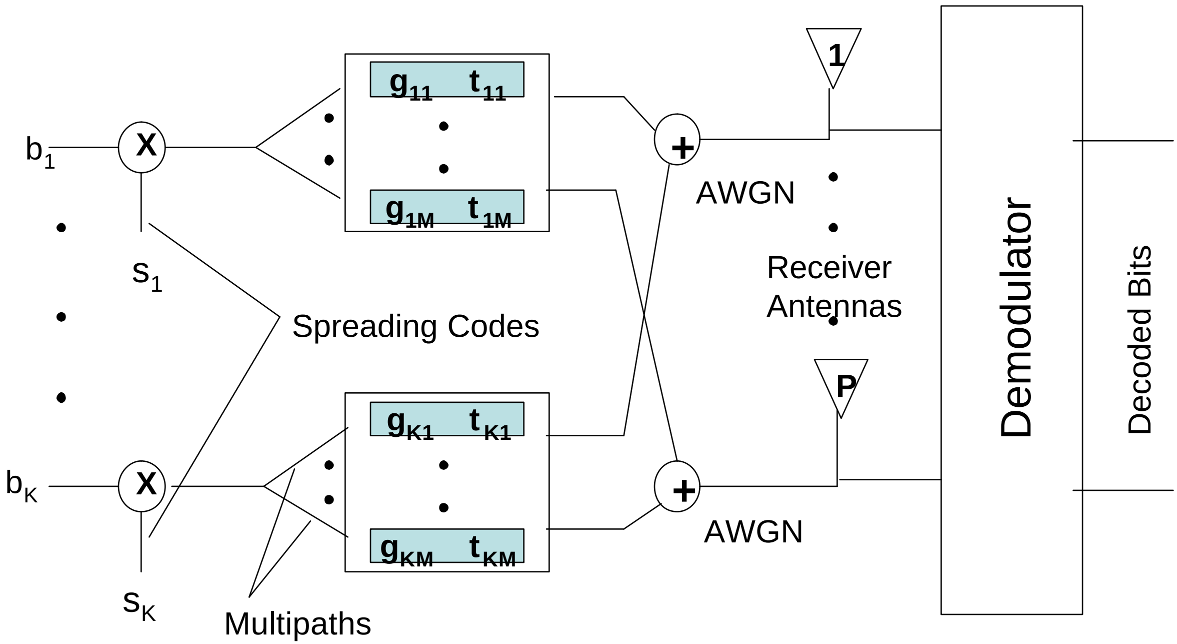
DS-CDMA Received Signal Model

Baseband received signal given by:

$$\mathbf{r}(\mathbf{t}) = \sum_i \sum_{k=1}^{K} A_k b_k(i) \sum_{m=1}^{M} \mathbf{a_{km}} g_{km} c_k(t - iT - \tau_{km}) + \sigma \mathbf{n}(\mathbf{t})$$

where:

- $b_k(i)$ – bit transmitted by the k^{th} user in the i^{th} interval, A_k – amplitude
- M – number of multipaths
- g_{km} & τ_{km} – complex gain & delay along m^{th} path of k^{th} user
- $\mathbf{a_{km}} = [a_{km,1}, \ldots, a_{km,P}]^H$ is the array response vector corresponding to m^{th} path of k^{th} user's signal with P receive antennas
- $c_k(t) = \frac{1}{\sqrt{N}} \sum_{n=1}^{N} s_{kn} \psi(t - (n-1)T_c)$, s_{kn} – spreading sequence
  - $\psi(t)$ – rectangular waveform with unit amplitude in $[0, T_c]$
- $\mathbf{n}(t) = [n_1(t), \ldots, n_P(t)]^H$ is a $P \times 1$ vector of zero mean complex Gaussian noise processes with unit variance
- $\mathbf{r}(t) = [r_1(t), \ldots, r_P(t)]^H$ is a $P \times 1$ vector



Replace t_{ij} by τ_{ij} for the channel delays

Discrete Time Signal Model

Sampling the received signal at the chip rate, we can write the discretised received signal vector:

$$\mathbf{r_p}(i) = \underbrace{\mathbf{S_{pL}}\mathbf{A}\mathbf{b}(i-1)}_{ISI} + \underbrace{\mathbf{S_{pR}}\mathbf{A}\mathbf{b}(i)}_{MAI} + \underbrace{\sigma\mathbf{n_p}}_{Noise}$$

where

$$\begin{aligned}
\alpha_{\mathbf{kp},\mathbf{L}} &= \sum_{m=1}^{M} a_{km,p} g_{km} \mathbf{s}_{\mathbf{kL}}^{(\mathbf{n_{km}})} \\
\alpha_{\mathbf{kp},\mathbf{R}} &= \sum_{m=1}^{M} a_{km,p} g_{km} \mathbf{s}_{\mathbf{kR}}^{(\mathbf{n_{km}})} \\
\mathbf{S_{pL}} &= [\alpha_{\mathbf{1p},\mathbf{L}} \ldots \alpha_{\mathbf{Kp},\mathbf{L}}] \quad (N \times K) \\
\mathbf{S_{pR}} &= [\alpha_{\mathbf{1p},\mathbf{R}} \ldots \alpha_{\mathbf{Kp},\mathbf{R}}] \quad (N \times K) \\
n_{km} &= \tau_{km}/T_c
\end{aligned}$$

Demodulation

- Let $\mathbf{w}_k$ denote FIR filter used to detect bits transmitted by the k^{th} user
- $\mathbf{w}_k$ operates on the augmented signal vector $\mathbf{r} = [\mathbf{r_1^H}, \ldots, \mathbf{r_P^H}]^H$ $(NP \times 1$ vector)
- Let $\mathbf{w_k} = [\mathbf{w_{k1}}^H, \ldots, \mathbf{w_{kP}}^H]^H$
- Soft output of filter for k^{th} user in i^{th} interval given by:

$$\begin{aligned} y_k(i) &= \mathbf{w_k^H r}(i) \\ &= \sum_{p=1}^{P} \mathbf{w_{kp}^H r_p}(i) \\ &= \sum_{p=1}^{P} \mathbf{w_{kp}^H S_{pL} A b}(i-1) + \sum_{p=1}^{P} \mathbf{w_{kp}^H S_{pR} A b}(i) + \sigma \mathbf{w_k^H n} \end{aligned}$$

- Bit decisions given by:

$$\widehat{b}_k(i) = sgn\left[\Re(y_k(i))\right]$$

Probability of Error based formulation

- Joint Probability of Error (P_E) can be expressed as :

$$P_E = 1 - \frac{1}{2^K} \sum_{\forall \mathbf{b}} \prod_{k=1}^{K} Q\left(-\frac{b_k \mu_k}{\sigma_k} \right)$$

where the conditional statistics,

$$\begin{aligned} \mu_k &= \Re\left(\sum_{p=1}^{P} \mathbf{w}_{\mathbf{kp}}^{\mathbf{H}} \mathbf{S}_{\mathbf{pL}} \mathbf{A} \mathbf{b}(i-1) + \sum_{p=1}^{P} \mathbf{w}_{\mathbf{kp}}^{\mathbf{H}} \mathbf{S}_{\mathbf{pR}} \mathbf{A} \mathbf{b} \right) \\ \sigma_k^2 &= \sigma^2 \|\mathbf{w}_{\mathbf{k}}\|^2 \end{aligned}$$

The problem is to design the filters $\{\mathbf{w}_{\mathbf{kp}}\}$ such that P_E is minimized.

- The complexity is $O(2^K)$ due to $\sum_{\mathbf{b}}$

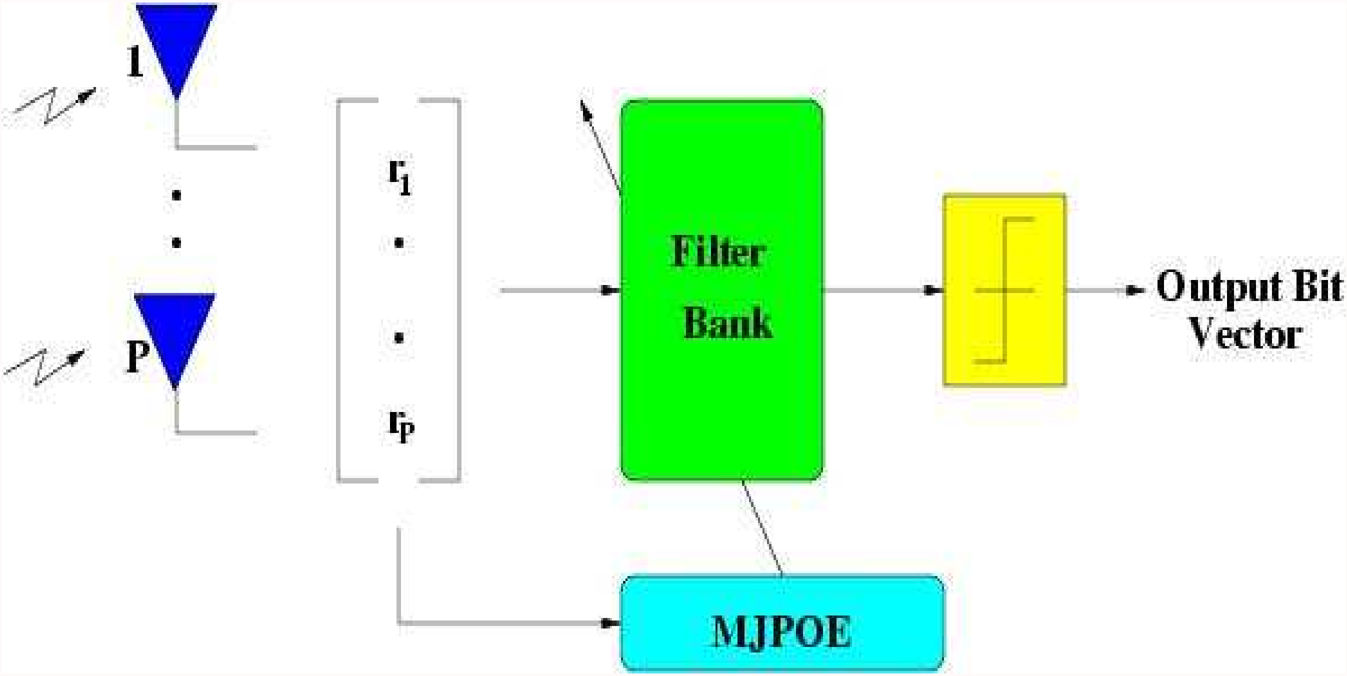


Figure 1: **Schematic of the MJPOE Adaptive Detector**

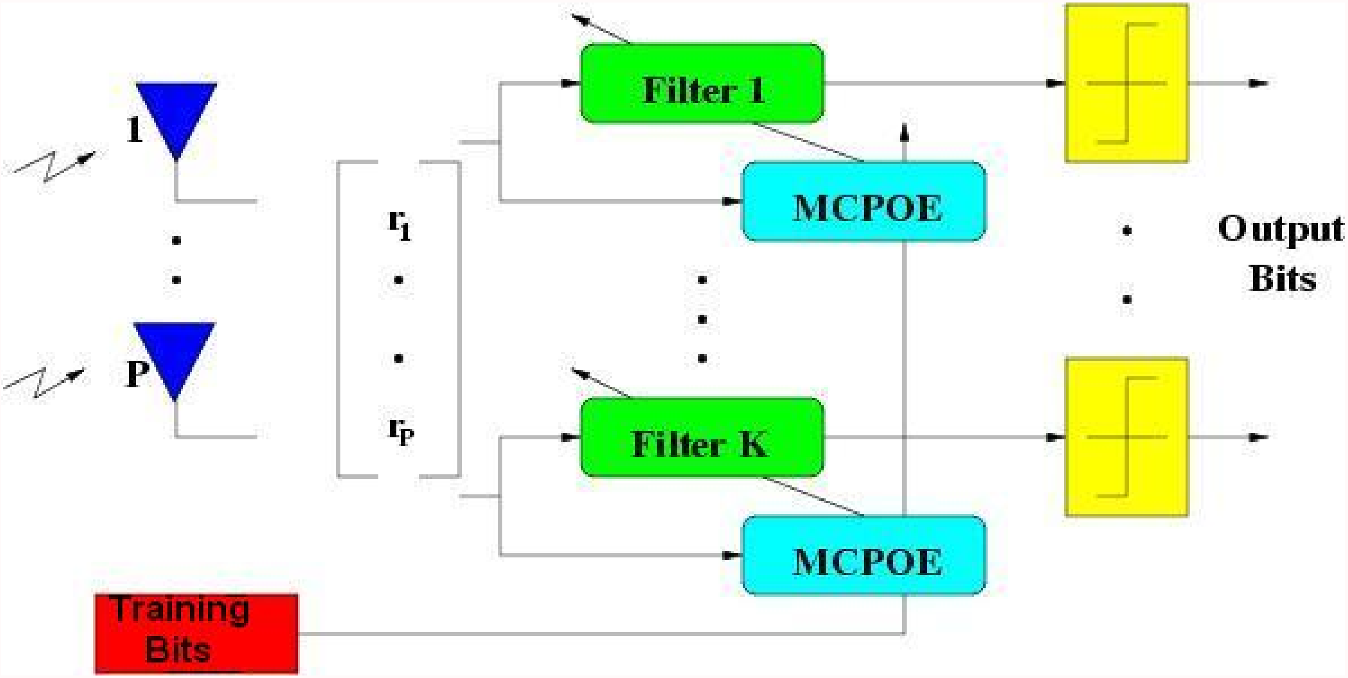


Figure 2: **Schematic of the MCPOE Adaptive Detector**

Efficient MJPOE

- Minimizes joint probability of error for all users
- **Estimate of the ISI term is subtracted to reduce the per symbol computation**
- Gives better BER performance than MJPOE
- Significant gains in computation

The Proposed Scheme

$$y_k(i) = \sum_{p=1}^{P} \mathbf{w_{kp}^H S_{pL} A b}(i-1) + \sum_{p=1}^{P} \mathbf{w_{kp}^H S_{pR} A b}(i) + \sigma \mathbf{w_k^H n}$$

- The first term in the original Decision Statistic y_k varies with each symbol.
- This requires weight updation for each symbol.
- Subtracting an estimate of this term from y_k, we obtain the new decision statistic z_k

$$z_k = y_k - \zeta_k$$

where the conditional statistics,

$$\zeta_k = \sum_{p=1}^{P} \mathbf{w}_{kp}^H \mathbf{S}_{pL} \mathbf{A} \hat{\mathbf{b}}(i-1)$$

- The new decision rule is

$$\hat{b}_k = sgn[\Re(z_k)]$$

- Joint Probability of Error (P_E) can now be expressed as :

$$\tilde{P}_E = 1 - \frac{1}{2^K} \sum_{\forall \mathbf{b}} \prod_{k=1}^{K} Q\left(-\frac{b_k \nu_k}{\sigma_k}\right)$$

where,

$$\begin{aligned} \nu_k &= \Re\left(\sum_{p=1}^{P} \tilde{\mathbf{w}}_{\mathbf{kp}}^{\mathbf{H}} \mathbf{S}_{\mathbf{pR}} \mathbf{A} \mathbf{b}\right) \\ \sigma_k^2 &= \sigma^2 \|\tilde{\mathbf{w}}_{\mathbf{k}}\|^2 \end{aligned}$$

The problem is to design the filters $\{\tilde{\mathbf{w}}_{\mathbf{kp}}\}$ such that $\tilde{P}_E$ is minimized.

- Gradient Descent technique is used for minimisation
- Though the complexity of the minimisation algorithm is $O(2^K)$ (due to $\sum_{\mathbf{b}}$), we can pre-compute the weights thereby reducing the computation per symbol

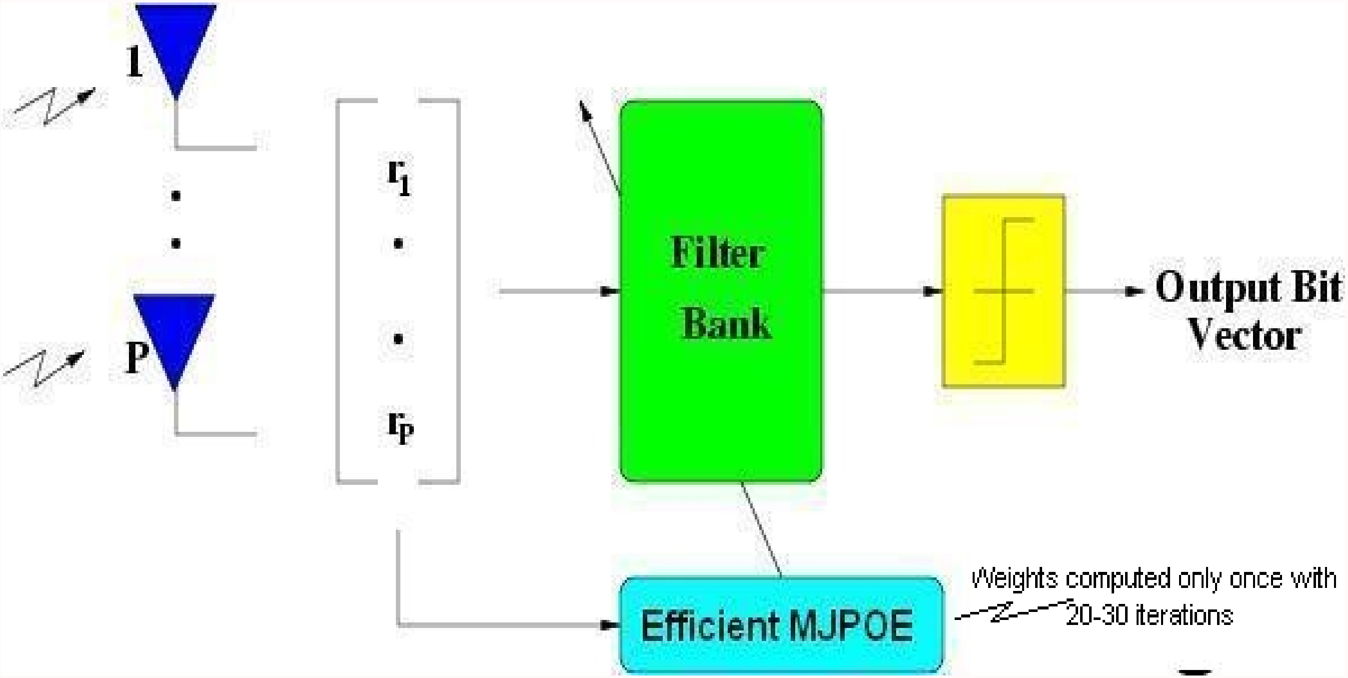


Figure 3: **Schematic of the Efficient MJPOE Adaptive Detector**

Some Remarks

- Subtract the estimate of the ISI term from the original decision statistic.
- Can pre-compute the weights since, unlike y_k, the statistics of z_k do not change with each symbol
- $E(z_k) = 0$, which is the decision threshold. Thus, reduction in BER is expected
- Simulation results support the claim

Error Propagation

- Direct subtraction of the estimated ISI term from the original decision statistic may lead to error propagation in case the previous decoded bit vector is in error.
- Simulations show that the system is robust to error and exhibits fast error recovery.

Simulation Results

- Efficient MJPOE compared with MJPOE both in convergence and BER performance
- Fully-known asynchronous frequency selective channels assumed for simulations
- The receiver was assumed to consist of a linear array of $P = 3$ antennas with half wavelength spacing and response given by $a_{km,p} = e^{j(p-1)\pi sin(\theta_{km})}$
- Multi-path gains ($U(0,1)e^{jU(-\pi/2,\pi/2)}$), DOA ($U(-\pi/2, \pi/2)$) and propagation delays ($\tau_k \leq T_{bit}\ \forall k$) were randomly generated for all users. $10,000$ bits were used to generate plots of the measured quantities.
- Error propagation simulated through deliberately inducing errors in the subtracted term

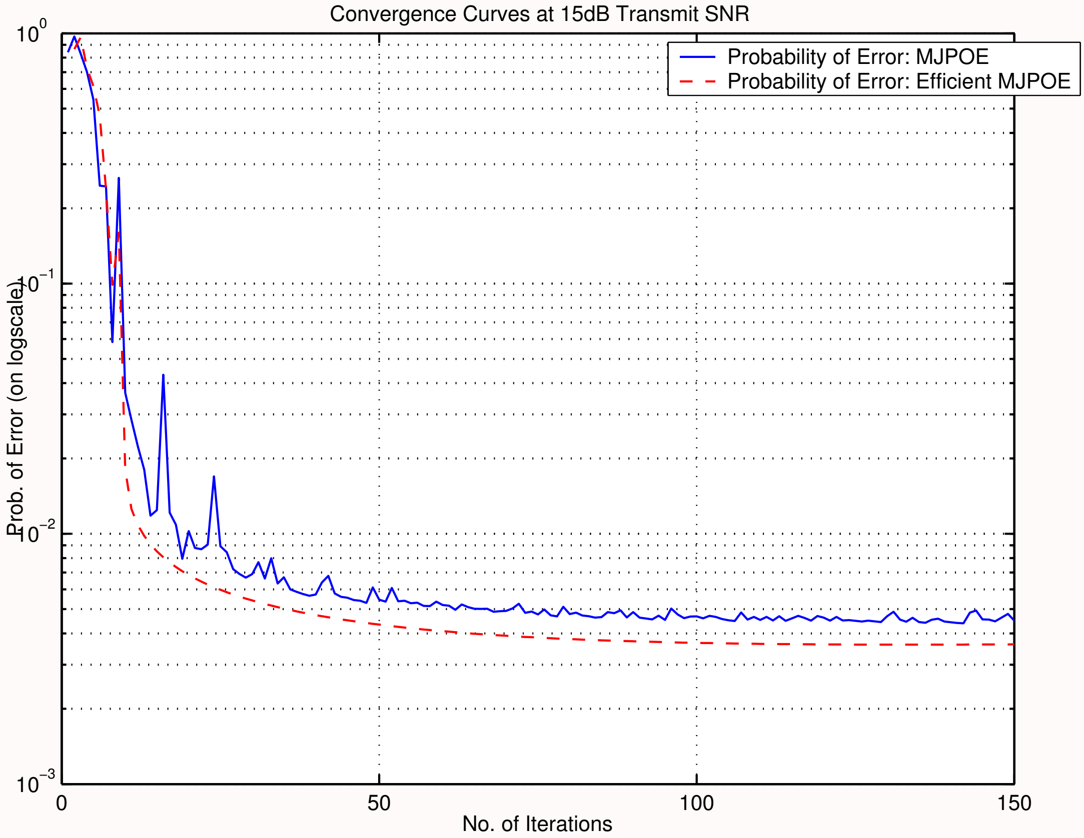


Figure 4: **Convergence performance comparison of MJPOE and Efficient MJPOE,** $K = 4, \quad M = 3, \quad N = 15, \quad P = 3$**. Plotted for a single case of MJPOE and Efficient MJPOE convergence**

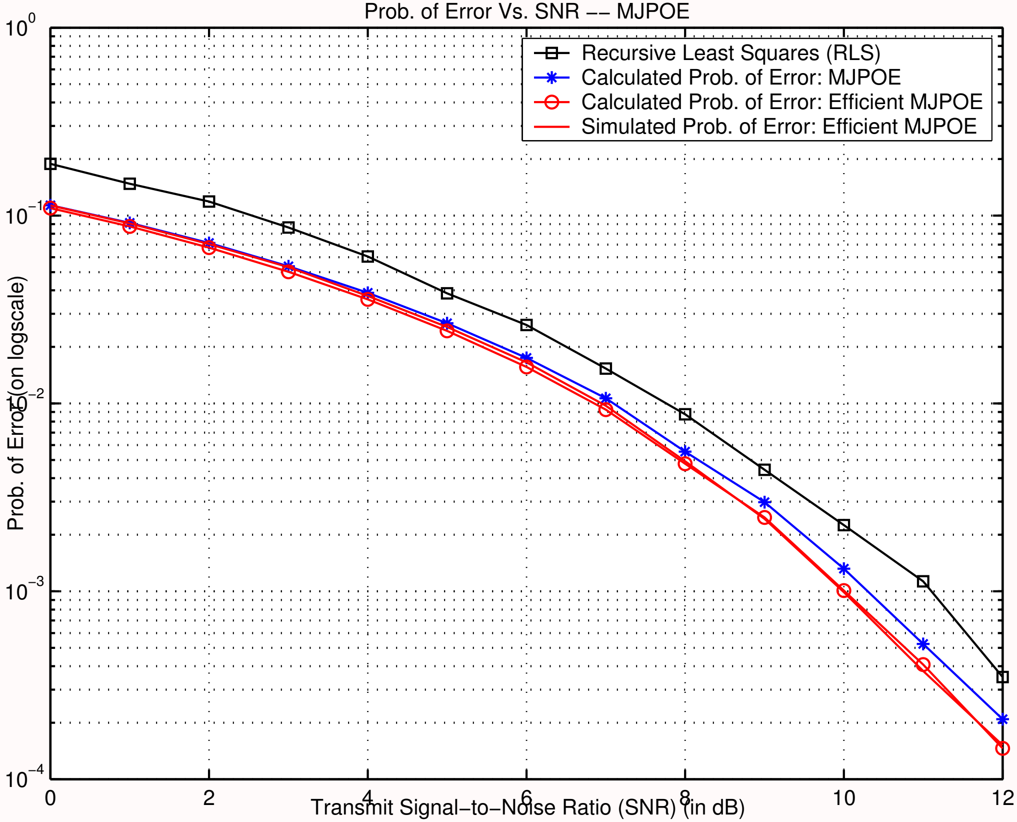


Figure 5: **BER performance comparison of MJPOE and Efficient MJPOE,** $K = 4, \ \ M = 3, \ \ N = 15, \ \ P = 3$**. Averaged over 15 independent channels**

Error Propagation Simulations

- Deliberate error introduced in all users
- Error exhibited in demodulated bits depending upon the ISI conditions of each user
- Fast error recovery observed in all cases

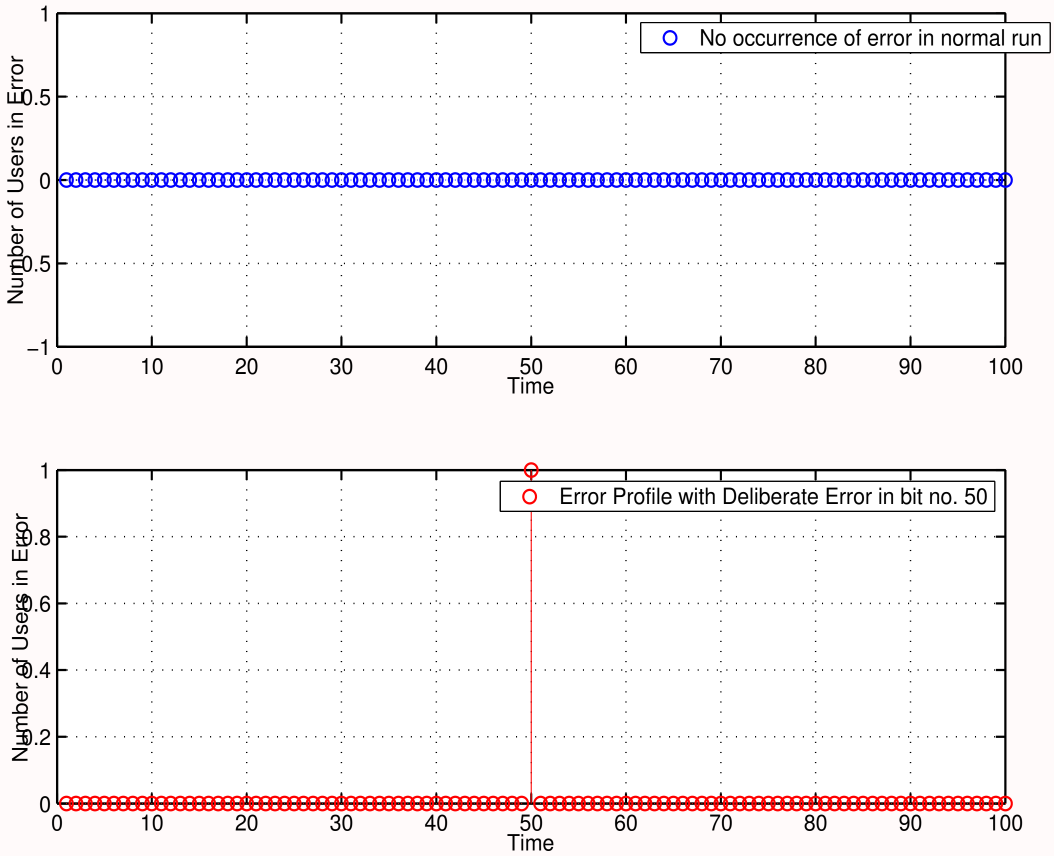


Figure 6: **Error propagation results in Efficient MJPOE due to deliberately induced error in ISI estimation. SNR=**$7dB$**.**

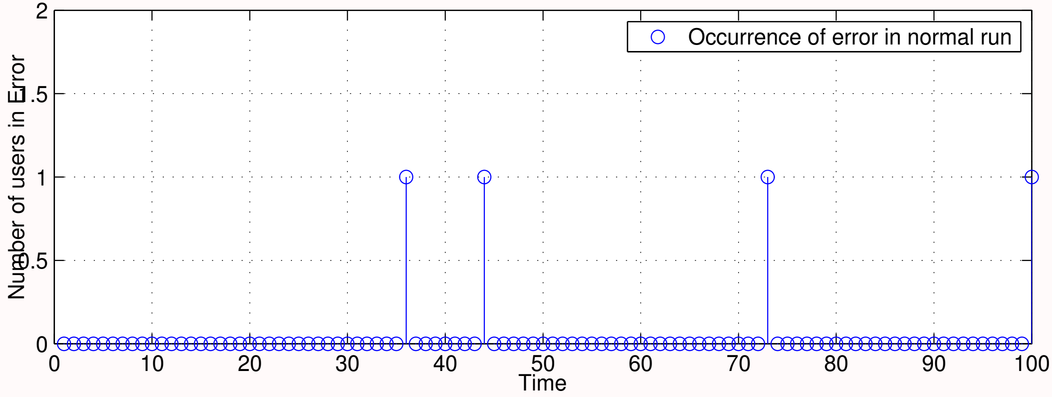


Figure 7: **Error recovery results in Efficient MJPOE due to consecutive induced errors in ISI estimation. SNR=$7dB$.**

Conclusion

- Efficient MJPOE performs better than MJPOE in terms of both convergence and BER
- High computational overheads reduced in Efficient MJPOE which allows pre-computation of weights
  - Hence, weight updation required only when the channel changes rather than at each symbol instant
- A similar modification applied to MCPOE also gives comparable performance.
- Excellent error recovery is observed during simulations

Many Thanks

$$\mathbf{s_{kL}^{(n_{km})}} = [\underbrace{s_{kN-n+1}, \ldots, s_{kN}}_{n_{km}} \;,\; \underbrace{0, \ldots, 0}_{N-n_{km}}]^T$$

$$\mathbf{s_{kR}^{(n_{km})}} = [\underbrace{0, \ldots, 0}_{n_{km}} \;,\; \underbrace{s_{k1}, \ldots, s_{kN-n}}_{N-n_{km}}]^T$$

MJPOE Adaptive Algorithm . . .

- ***Observation :*** From simulations it was observed that at convergence filters of various users were nearly orthogonal to each other, i.e., $\mathbf{w}_k^H \mathbf{w}_l \approx 0$
- Intuitively expected. We want the filters to be well separated, or else they will detect the same user
- Can be incorporated as apriori into the optimization problem
- **If filters are orthogonal, $\mathbf{C}$ becomes a diagonal matrix - the decision statistics of various users y_k become uncorrelated**

Prof. U. B. Desai
ubdesai@ee.iitb.ac.in
Dept. of Electrical Engineering
Indian Institute of Technology - Bombay
Mumbai 400076