MPOE Prefiltering with Statistical Channel Model for DS-CDMA Systems

G. Kannan, Mohit Garg†, S. N. Merchant and U. B. Desai
SPANN Laboratory, Department of Electrical Engineering
Indian Institute of Technology-Bombay, Mumbai, India-400076
Email: {gkannan, merchant, ubdesai}@ee.iitb.ac.in, †mohitgarg@gmail.com

Abstract— **In order to reduce the complexity of the mobile receiver, we develop a linear precoding filter based only on the statistical knowledge of the channel. Moreover, the proposed prefilter (precoder) is based on minimizing the probability of error in downlink multiuser transmission. We investigate two approaches for the proposed algorithm. In the first approach we consider a common FIR precoding filter for all users, and jointly minimize the probability of error of all users. In the second approach, we assume separate precoders for each user which are obtained by minimizing the probability of error for the respective user. In order to fully utilize the knowledge available at the transmitter, in both the approaches, the filter weights are computed conditioned on the transmitted bit vector sequence. This also makes the computation of the optimal prefilter coefficients linear in the number of users. We compare the results of the proposed approach, with results based on assuming complete knowledge of the channel. Simulation results clearly show that precoders based only on the statistical knowledge of channel, do provide acceptable BERs. Moreover, individual precoders provide better BER as compared to joint precoders.**

I. INTRODUCTION

In this paper, we explore a prefiltering scheme at the base station transmitter which allows considerably simplified receiver structure in downlink transmission. Prefilter design is simple when the transmitter has complete channel knowledge. Unfortunately, in practice it is highly improbable to have full channel knowledge. Even in Time Division Duplex (TDD) systems, in fast fading channel conditions, it is extremely difficult to keep track of the channel variations with time. In this paper, we explore the approach of working only with first order and second order statistics of the channel at the transmitter and optimizing the transmitter precoding filter. The basic assumption is that the statistics of the channel change at a much slower rate than the channel itself and hence it may be reasonable to assume that tracking the channel statistics will be easier to implement in practical systems. Our approach becomes more attractive because the channel statistics for the most commonly used channels in wireless mobile communications have been well studied and standard models are available for the same [1]-[2]. We consider two approaches for prefiltering: the first one considers the common prefilter for all users as shown in Fig. 1. In the second approach, we consider a system which has an individual prefilter for each user as shown in Fig. 2. The standard single user receiver (conventional matched filter detector) is used in both of our models. Minimum Mean Squared Error (MMSE) has been traditionally used as the optimization criterion for precoder/detector design in many wireless systems. We believe since symbols are of significance in digital communications, optimum prefilter design should be based on minimizing the probability of symbol error at the receiver. We refer to this as Minimum Probability of Error (MPOE) prefilter [3]-[4]. Usually MPOE optimization tends to be computationally expensive, but as we assume ample computational resources at the base station, using MPOE instead of MMSE as the optimization criterion can be justified. Moreover, by conditioning the filter weights on the transmitted bits, one can design an MPOE precoder with linear complexity [3]-[4].

The rest of the paper is organized as follows: Section II briefly describes the related work. System model is introduced in Section III. MPOE and MMSE based joint prefilters are derived in Section IV. Individual prefilterings with MPOE and MMSE optimizations are discussed in Section V. Simulation results and discussions are provided in Section VI. Finally some concluding remarks are given in Section VII.

II. RELATED WORK

Significant research has been carried out in the area of prefiltering over the last few years. However almost all of it has been directed towards the design of MMSE based prefilters; moreover most of the approaches assume complete knowledge of the forward channel [5]-[8]. MPOE optimization method was proposed by Dua *et.al.* in [3]-[4] and later extended to decision feedback detector in [9]. Furthermore in [3]-[4], a linear computational complexity receiver with respect to number of users, was proposed. In [3]-[4] it was established that MPOE optimization has better performance than MMSE optimization. Independently, Minimum Bit Error Rate (MBER) optimization for decision feedback equalizer receiver was proposed in [10]. In [11], Ding *et. al.* proposed a MBER precoder and in [12] Palomar *et. al.* derived a transceiver based on MBER method for Zero-Forcing (ZF) equalizer at the receiver. These systems require a complex receiver model with complete channel knowledge and precoder knowledge. Georgoulis *et. al.* in [7] and Reynolds *et. al.* in [8] have used a simple matched filter receiver by considering a channel model with ISI, but the optimization criterion is MMSE. MPOE based prefiltering by considering a general channel model (MAI+ISI) and simple matched filter receiver was first proposed in [13]. All the above papers assume complete channel knowledge. Precoder

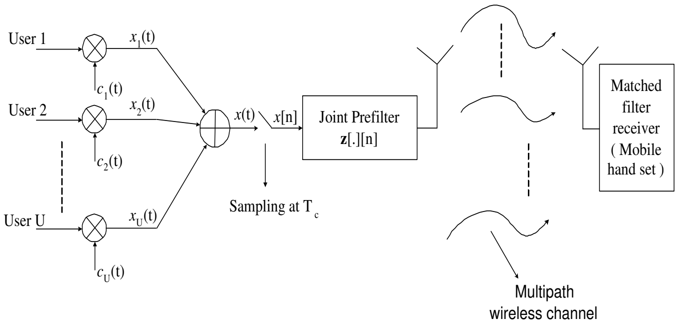

Fig. 1. DS-CDMA channel model for a multipath channel with joint transmitter prefiltering

design based on partial channel knowledge have been given in [14]-[18]. However, to the best of our knowledge, there is no treatment on MPOE based precoding with minimal complexity receivers, by considering a general channel model (MAI+ISI) and assuming only statistical knowledge of the channel.

III. SIGNAL MODEL

Consider a DS-CDMA system with U users as shown in Fig. 1. The base station transmits BPSK bit $b_u(i)$, with amplitude A_u, in ith interval, for user u. The length of signaling interval for each user is T_{bit}. Assume that user u is assigned a spreading waveform $c_u(.)$ and $\mathbf{s}_u = [s_{u0}, s_{u1}, \ldots, s_{uN-1}]^T$ denotes the corresponding spreading sequence. Then,

$$c_u(t) = \sum_{k=0}^{N-1} s_{uk} rect[t - kT_c], \qquad u = 1, 2, \ldots, U$$

where, T_c is the chip period, $rect(t)$ is a rectangular waveform with unit amplitude in [0, T_c] and N is the processing gain of the system. The baseband signal for the uth user in the ith bit interval can now be expressed as [19]-[20]

$$x_u(t) = A_u b_u(i) c_u(t - iT_{bit}), \qquad iT_{bit} \leq t < (i+1)T_{bit}$$

By summing up all the users' signal at ith bit interval, we get

$$x(t) = \sum_{u=1}^{U} A_u b_u(i) c_u(t - iT_{bit}), \qquad iT_{bit} \leq t < (i+1)T_{bit}$$

Assume that $x(t)$ is sampled $(x[n])$ at T_c (chip rate sampling) [20], then

$$x[n] = \sum_{u=1}^{U} A_u b_u(n_N) \tilde{s}_u[n] \tag{1}$$

where $n_N = \lfloor \frac{n}{N} \rfloor$ because of chip rate sampling (note that $NT_c = T_{bit}$) and $\tilde{s}_u[n] = s_{u((n)mod(N))}$ [20].

We assume that the prefilter for a particular bit period i is a FIR filter of length L_z ($\mathbf{z}[.][i]$) and it will be calculated adaptively at every bit interval. The idea is to compute these filter coefficients using MPOE and MMSE criteria. The prefiltered signal that will be sent through the wireless channel is

$$x[n] \otimes \mathbf{z}[.][n]$$

where $\otimes$ denotes the convolution operation and $\mathbf{z}[.][n]$ is the prefilter at time instant n. A general multipath frequency selective channel is assumed in our system. The multipath wireless channel is modeled as a FIR filter and the channel coefficients are assumed to be constant over one bit period. The channel for the uth user at the ith bit interval is denoted as $\mathbf{h}_u[.][i]$ which is of length L_h^u for all i.

The elements of the channel FIR filter ($h_u[l][n]$) are assumed to be complex Gaussian with both real and imaginary parts follow $i.i.d$ Gaussian. The noise ($\eta_u[n]$) is assumed to be $i.i.d$ zero mean additive white Gaussian (AWGN). The received signal at user u is

$$r_u[n] = \mathbf{h}_u[.][n] \otimes x[n] \otimes \mathbf{z}[.][n] + \eta_u[n] \tag{2}$$

where $\mathbf{h}_u[.][n]$ is the channel FIR filter at time instant n. Converting $r_u[n]$ into a parallel stream of N samples (number of chips), we obtain

$$\mathbf{r}_u[i] = [r_u[iN], \ldots, r_u[iN + N - 1]]^T$$

We use a simple matched filter receiver. The received signal at the uth user after matched filtering is

$$y_u[i] = \mathbf{s}_u^T \mathbf{r}_u[i] = \sum_{k=0}^{N-1} s_{uk} r_u[iN + k] \tag{3}$$

From (2) and (3)

$$\begin{aligned} y_u[i] = \sum_{k=0}^{N-1} s_{uk} \sum_{m=0}^{L_h^u - 1} h_u[m][j_1] \sum_{l=0}^{L_z - 1} z[l][j_2] \sum_{v=1}^{U} A_v b_v[j_2]. \\ .\tilde{s}_v[iN + k - m - l] + \sum_{k=0}^{N-1} s_{uk} \eta_u[iN + k] \end{aligned} \tag{4}$$

where

$$j_1 = \left\lfloor \frac{iN + k - m}{N} \right\rfloor, \quad j_2 = \left\lfloor \frac{iN + k - m - l}{N} \right\rfloor \tag{5}$$

In (4), $\sum_{m=0}^{L_h^u - 1}$ gives the ISI term due to the multipath channel while $\sum_{v=1}^{U}$ is the MAI component due to multiple user transmission with correlated spreading codes. Since BPSK constellations are used for input data, the decision statistics is given by $\Re(y_u[i]) = y_u^R[i]$.

IV. PROPOSED JOINT PREFILTERING MODEL

We assume only the statistical parameters namely, mean and variance of the channel coefficients are available at the transmitter and derive the MPOE/MMSE based prefilterings. We first compute the conditional probability of error ($P_{E|\mathbf{B}[i]}$) conditioned on transmitted bit vector sequence $\mathbf{B}[i] = \mathbf{b}[i], \mathbf{b}[i-1], \ldots$, where $\mathbf{b}[i] = \left[b_1[i], b_2[i], \ldots, b_U[i]\right]^T$ is the vector of bits transmitted at time instant i for all the users.

A. MPOE based prefiltering

Let us define the mean of the mth channel coefficient at ith bit interval as

$$\gamma_u[m][i] = E\Big[h_u[m][i]\Big] \tag{6}$$

and the second order statistics of the channel coefficients as

$$\begin{aligned}
C_u[m_1,m_2][i_1,i_2] = E\Bigg[&\Big(h_u[m_1][i_1]-\gamma_u[m_1][i_1]\Big).\\
&.\Big(h_u^*[m_2][i_2]-\gamma_u^*[m_2][i_2]\Big)\Bigg]
\end{aligned}$$

$$\begin{aligned}
\tilde{C}_u[m_1,m_2][i_1,i_2] = E\Bigg[&\Big(h_u[m_1][i_1]-\gamma_u[m_1][i_1]\Big).\\
&.\Big(h_u[m_2][i_2]-\gamma_u[m_2][i_2]\Big)\Bigg]
\end{aligned} \tag{7}$$

$$\begin{aligned}
R_u[m_1,m_2][i_1,i_2] &= E\Big[h_u[m_1][i_1]h_u^*[m_2][i_2]\Big]\\
\tilde{R}_u[m_1,m_2][i_1,i_2] &= E\Big[h_u[m_1][i_1]h_u[m_2][i_2]\Big]
\end{aligned} \tag{8}$$

where * denotes complex conjugate. The conditional mean ($\mu_{y_u^R|\mathbf{B}[i]}$) of the decision statistic ($y_u^R[i]$) is given by

$$\begin{aligned}
\mu_{y_u^R|\mathbf{B}[i]}[i] = E\Bigg[\Re\Bigg(&\sum_{k=0}^{N-1} s_{uk}\sum_{m=0}^{L_h^u-1} h_u[m][j_1]\sum_{l=0}^{L_z-1} z[l][j_2].\\
&.\sum_{v=1}^{U} A_v b_v[j_2]\tilde{s}_v[iN+k-m-l]+\sum_{k=0}^{N-1} s_{uk}\eta_u[iN+k]\Bigg)\Bigg]
\end{aligned}$$

By using the fact that $E[\Re(a)] = \Re(E[a])$ for any a, $E(\eta_u[iN+k]) = 0$, and except $h_u[m][j_1]$, $\eta_u[iN+k]$ all other quantities are deterministic, the above equation can be written as

$$\begin{aligned}
\mu_{y_u^R|\mathbf{B}[i]}[i] = \Re\Bigg[&\sum_{k=0}^{N-1} s_{uk}\sum_{m=0}^{L_h^u-1} \gamma_u[m][j_1]\sum_{l=0}^{L_z-1} z[l][j_2].\\
&.\sum_{v=1}^{U} A_v b_v[j_2]\tilde{s}_v[iN+k-m-l]\Bigg]
\end{aligned} \tag{9}$$

The conditional variance of the decision statistics ($\sigma^2_{y_u^R|\mathbf{B}[i]}$) is

$$\begin{aligned}
\sigma^2_{y_u^R|\mathbf{B}[i]}[i] = var\Bigg[\Re\Bigg(&\sum_{k=0}^{N-1} s_{uk}\sum_{m=0}^{L_h^u-1} h_u[m][j_1]\sum_{l=0}^{L_z-1} z[l][j_2].\\
&.\sum_{v=1}^{U} A_v b_v[j_2]\tilde{s}_v[iN+k-m-l]\Bigg)\Bigg]+N\frac{\sigma^2}{2}
\end{aligned} \tag{10}$$

where σ^2 is the variance of AWGN noise. Let

$$\begin{aligned}
\sigma^2_{\tilde{y}_u^R|\mathbf{B}[i]}[i] = var\Bigg[\Re\Bigg(&\sum_{k=0}^{N-1} s_{uk}\sum_{m=0}^{L_h^u-1} h_u[m][j_1]\sum_{l=0}^{L_z-1} z[l][j_2].\\
&.\sum_{v=1}^{U} A_v b_v[j_2]\tilde{s}_v[iN+k-m-l]\Bigg)\Bigg]
\end{aligned} \tag{11}$$

and this can be rewritten as

$$\begin{aligned}
\sigma^2_{\tilde{y}_u^R|\mathbf{B}[i]}[i] = var\Bigg[\Re\Bigg(&\sum_{k=0}^{N-1}\sum_{m=0}^{L_h^u-1}\sum_{l=0}^{L_z-1}\sum_{v=1}^{U} s_{uk}h_u[m][j_1]z[l][j_2].\\
&.A_v b_v[j_2]\tilde{s}_v[iN+k-m-l]\Bigg)\Bigg]
\end{aligned}$$

This can be further rewritten as

$$\sigma^2_{\tilde{y}_u^R|\mathbf{B}[i]}[i] = var\Bigg[\Re\Bigg(\sum_{\substack{k,m,\\ l,u}} h_u[m][j_1]f_u[k,m,l,v]\Bigg)\Bigg] \tag{12}$$

where $f_u[k,m,l,v] = s_{uk}z[l][j_2]A_v b_v[j_2]\tilde{s}_v[iN+k-m-l]$ is deterministic variable, the indices j_1, j_2 are given in (5) and the sum

$$\sum_{\substack{k,m,\\ l,v}} = \sum_{k=0}^{N-1}\sum_{m=0}^{L_h^u-1}\sum_{l=0}^{L_z-1}\sum_{v=1}^{U}$$

Since $f_u[k,m,l,v]$ is deterministic variable and by using the fact that $var(a) = E[a-E(a)]^2$, (12) can be written as $\sigma^2_{\tilde{y}_u^R|\mathbf{B}[i]}[i]$

$$\begin{aligned}
&= E\Bigg[\Bigg(\Re\Bigg[\sum_{\substack{k,m,\\ l,v}}(h_u[m][j_1]-\gamma_u[m][j_1])f_u[k,m,l,v]\Bigg]\Bigg)^2\Bigg]\\
&= E\Bigg[\Re\Bigg(\sum_{\substack{k_1,m_1,\\ l_1,v_1}}(h_u[m_1][j_{11}]-\gamma_u[m_1][j_{11}])f_u[k_1,m_1,l_1,v_1]\Bigg).\\
&\quad .\Re\Bigg(\sum_{\substack{k_2,m_2,\\ l_2,v_2}}(h_u[m_2][j_{21}]-\gamma_u[m_2][j_{21}])f_u[k_2,m_2,l_2,v_2]\Bigg)\Bigg]
\end{aligned} \tag{13}$$

where,

$$j_{11} = \left\lfloor\frac{iN+k_1-m_1}{N}\right\rfloor, \qquad j_{21} = \left\lfloor\frac{iN+k_2-m_2}{N}\right\rfloor \tag{14}$$

For notational convenience let us assume

$$\begin{aligned}
&h_1 = h_u[m_1][j_{11}],\ \gamma_1 = \gamma_u[m_1][j_{11}],\ f_1 = f_u[k_1,m_1,l_1,v_1]\\
&h_2 = h_u[m_2][j_{21}],\ \gamma_2 = \gamma_u[m_2][j_{21}],\ f_2 = f_u[k_2,m_2,l_2,v_2]
\end{aligned} \tag{15}$$

From (13), (15)

$$\sigma^2_{\tilde{y}_u^R|\mathbf{B}[i]}[i] = E\Bigg[\sum_{\substack{k_1,m_1,l_1,v_1,\\ k_2,m_2,l_2,v_2}} \Re[(h_1-\gamma_1)f_1]\Re[(h_2-\gamma_2)f_2]\Bigg]$$

By using the fact that $\Re(a) = \frac{(a+a^*)}{2}$ for any complex number a,

$$\sigma^2_{\tilde{y}^R_u|\mathbf{B}[i]}[i] = \frac{1}{4}E\Bigg[\sum_{\substack{k_1,m_1,l_1,v_1,\\ k_2,m_2,l_2,v_2}}\Big(h_1f_1-\gamma_1f_1+h_1^*f_1^*-\gamma_1^*f_1^*\Big)\cdot \Big(h_2f_2-\gamma_2f_2+h_2^*f_2^*-\gamma_2^*f_2^*\Big)\Bigg]$$

Since f_1 and f_2 are deterministic the above equation will become

$$\sigma^2_{\tilde{y}^R_u|\mathbf{B}[i]}[i] = \frac{1}{4}\Bigg[\sum_{\substack{k_1,m_1,l_1,v_1,\\ k_2,m_2,l_2,v_2}}\tilde{C}_uf_1f_2+\tilde{C}_u^*f_1^*f_2^*+C_uf_1f_2^* + C_u^*f_1^*f_2\Bigg] \tag{16}$$

where $C_u = C[m_1,m_2][j_{11},j_{21}]$ and $\tilde{C}_u = \tilde{C}[m_1,m_2][j_{11},j_{21}]$ can be calculated using (7). From (10), (11) and (16) the conditional variance will become

$$\sigma^2_{y^R_u|\mathbf{B}[i]}[i] = \frac{1}{4}\Bigg[\sum_{\substack{k_1,m_1,l_1,v_1,\\ k_2,m_2,l_2,v_2}}\tilde{C}_uf_1f_2+\tilde{C}_u^*f_1^*f_2^*+C_uf_1f_2^* + C_u^*f_1^*f_2\Bigg]+N\frac{\sigma^2}{2}$$

Now the conditional probability of error is

$$P_{E|\mathbf{B}[i]}[i] = Q\left(\frac{b_u[i]\mu_{y^R_u|\mathbf{B}[i]}[i]}{\sigma_{y^R_u|\mathbf{B}[i]}[i]}\right) \tag{17}$$

and the corresponding probability of correct detection is

$$Q\left(-\frac{b_u[i]\mu_{y^R_u|\mathbf{B}[i]}[i]}{\sigma_{y^R_u|\mathbf{B}[i]}[i]}\right) \tag{18}$$

by the fact that $1-Q(x) = Q(-x)$.

We would like to minimize the joint conditional probability of error for all users, namely,

$$P_{EJ}[i] = 1 - P[y^R_1 \in \alpha_1, y^R_2 \in \alpha_2, \dots, y^R_U \in \alpha_U]$$

where $P[y^R_u \in \alpha_u]$ is the probability of correct detection for the uth user, α_u is the decision region for symbol detection for the uth user and J denotes joint probability of error. We have omitted the conditioning markers and index i for notational ease. Since the noise vectors for all users are independent of each other, the joint conditional probability of error becomes

$$P_{EJ}[i] = 1 - P[y^R_1 \in \alpha_1]P[y^R_2 \in \alpha_2]\dots P[y^R_U \in \alpha_U] \tag{19}$$

Using (18), (19) and since we have assumed identical distribution assumption for noise, P_{EJ} can be written in closed form as

$$P_{EJ}[i] = 1 - \prod_{u=1}^{U} Q\left(-\frac{b_u[i]\mu_{y^R_u|\mathbf{B}[i]}[i]}{\sigma_{y^R_u|\mathbf{B}[i]}[i]}\right)$$

The prefilter coefficients ($\mathbf{z}[.][i]$) of length L_z for each bit interval i is calculated by minimizing the above formulated probability of error. A stochastic gradient descent approach can now be used to determine the prefilter coefficients as follows

$$\mathbf{z}[.][i+1] = \mathbf{z}[.][i] - \mu\frac{\partial P_{EJ}}{\partial \mathbf{z}[.][i]} \tag{20}$$

where μ is an appropriately chosen step-size parameter, and in general it could be adaptive.

B. MMSE based prefiltering

Let y^R_u be the decision statistic for the uth user. Thus, at every bit period i, we get a vector $\mathbf{y}^R[i] = \Big[y^R_1[i],\ y^R_2[i], y^R_3[i], \dots, y^R_U[i]\Big]^T$ of decision statistics for all users. Therefore the cost function for MMSE based algorithm is

$$\xi^2_{J|\mathbf{B}[i]}[i] = E\Big[||(\mathbf{y}^R_u[i] - \mathbf{b}[i])||^2|\mathbf{B}[i]\Big]$$

Let us drop index i for notational convenience

$$\begin{aligned}\xi^2_{J|\mathbf{B}[i]} &= \sum_{u=1}^{U} E\Big[((y^R_u)^2 + b_u^2 - 2y^R_u b_u)|\mathbf{B}\Big]\\ &= \sum_{u=1}^{U}\Big[E((y^R_u)^2|\mathbf{B}) + 1 - 2b_uE(y^R_u|\mathbf{B})\Big]\end{aligned} \tag{21}$$

Since $\sigma^2_{\tilde{y}^R_u|\mathbf{B}[i]} = E((y^R_u)^2|\mathbf{B})$ when mean is zero, the same procedure for calculation of $\sigma^2_{\tilde{y}^R_u|\mathbf{B}[i]}[i]$ can be followed to find out $E((y^R_u)^2|\mathbf{B})$ except that R_u, $\tilde{R}_u$ replace C_u, $\tilde{C}_u$ (because C_u, $\tilde{C}_u$ become R_u, $\tilde{R}_u$ when mean is zero). Hence $E((y^R_u)^2|\mathbf{B})$ can be written as

$$E((y^R_u)^2|B) = \frac{1}{4}\Bigg[\sum_{\substack{k_1,m_1,l_1,u_1,\\ k_2,m_2,l_2,u_2}}\tilde{R}_uf_1f_2+\tilde{R}_u^*f_1^*f_2^*+R_uf_1f_2^* + R_u^*f_1^*f_2\Bigg] \tag{22}$$

where $\tilde{R}_u = \tilde{R}_u[m_1,m_2][j_{11},j_{21}]$, $R_u = R_u[m_1,m_2][j_{11},j_{21}]$ can be calculated using (8). From (21) and (22) the cost function is

$$\xi^2_{J|\mathbf{B}[i]}[i] = \sum_{u=1}^{U}\Bigg[\frac{1}{4}\Bigg(\sum_{\substack{k_1,m_1,l_1,u_1,\\ k_2,m_2,l_2,u_2}}\tilde{R}_uf_1f_2+\tilde{R}_u^*f_1^*f_2^*+R_uf_1f_2^* + R_u^*f_1^*f_2\Bigg)+1-2b_u[i]\mu_{y^R_u|\mathbf{B}[i]}\Bigg]$$

where $\mu_{y^R_u|\mathbf{B}[i]}$ is given by (9). We follow a similar stochastic gradient descent approach as in (20) to compute $\mathbf{z}[.][i]$ by minimizing $\xi^2_{J|\mathbf{B}[i]}[i]$.

Note that in the statistical channel algorithm, we only need the knowledge of the first order ($\gamma_u[.][.]$) and second order ($C_u[.,.][.,.], \dots, C_u[.,.][.,.]$) statistics of the channel coefficients $\mathbf{h}_u[.][i]$.

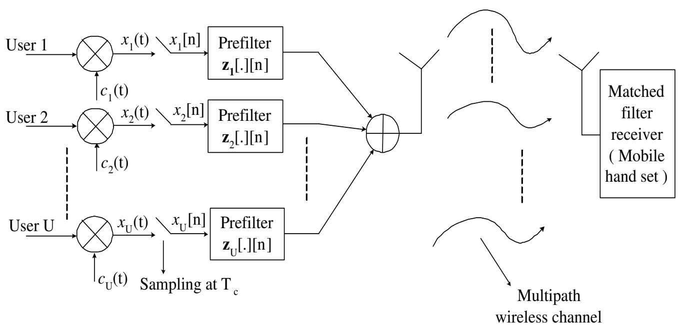

Fig. 2. DS-CDMA channel model for a multipath channel with individual prefiltering

V. PROPOSED INDIVIDUAL PREFILTERING MODEL

In this model the data for user u after being spread is prefiltered by a FIR filter of length L_z^u with a discrete time impulse response $\mathbf{z}_u[.][i]$ as shown in Fig. 2. The resulting modified signals are summed to form the final transmitted signal. The prefilters $\mathbf{z}_u[.][i]$, $u = 1, \ldots, U$ are designed such that the probability of error for that particular user is minimum at the receiver. Each user's prefilter is designed individually by taking into account the channel information and the transmit codes of that particular user. The signal for user u in base station is

$$x_u(t) = A_u b_u(i) c_u(t - iT_{bit}), \qquad iT_{bit} \leq t < (i+1)T_{bit}$$

This signal is sampled at chip rate as explained in (1) and will be processed through a prefilter. The prefiltered signal at time instant n corresponding to user u is given by

$$x_u[n] \otimes \mathbf{z}_u[.][n]$$

Now the transmitted signal is given by

$$\sum_{u=1}^{U} \Big(x_u[n] \otimes \mathbf{z}_u[.][n] \Big)$$

The received signal at user u is

$$r_u[n] = \mathbf{h}_u[.][n] \otimes \sum_{u=1}^{U} \Big(x_u[n] \otimes \mathbf{z}_u[.][n] \Big) + \eta_u[n]$$

If we carry out the same formulation as in (2)-(4), the received signal at user u after matched filtering is

$$y_u[i] = \Bigg[\sum_{k=0}^{N-1} s_{uk} \sum_{m=0}^{L_h^u - 1} h_u[m][j_1] \sum_{l=0}^{L_z^u - 1} \sum_{v=1}^{U} z_v[l][j_2] A_v b_v[j_2] . \tilde{s}_v[iN + k - m - l] \Bigg] + \sum_{k=0}^{N-1} s_{uk} \eta_u[iN + k] \tag{23}$$

where j_1 and j_2 are given by (5).

A. *MPOE based prefiltering*

The probability of error for a user u ($P^u_{E|\mathbf{B}[i]}[i]$) can be formulated in the same way as given in (17), except the fact that the prefilter is different for each user. By analysing along the similar lines as of (6)-(17) and by replacing $y_u[i]$ in (6)-(17) by (23), the conditional mean ($\mu_{y_u^R|\mathbf{B}[i]}$) and the conditional variance ($\sigma^2_{y_u^R|\mathbf{B}[i]}$) can be derived as

$$\mu_{y_u^R|\mathbf{B}[i]}[i] = \Re \Bigg[\sum_{k=0}^{N-1} s_{uk} \sum_{m=0}^{L_h^u - 1} \gamma_u[m][j_1] \sum_{l=0}^{L_z^u - 1} \sum_{v=1}^{U} z_u[l][j_2] . A_v b_v[j_2] \tilde{s}_v[iN + k - m - l] \Bigg]$$

$$\sigma^2_{y_u^R|\mathbf{B}[i]}[i] = var \Bigg(\Re \Bigg[\sum_{k=0}^{N-1} s_{uk} \sum_{m=0}^{L_h^u - 1} h_u[m][j_1] \sum_{l=0}^{L_z^u - 1} \Big(\sum_{v=1}^{U} z_v[l][j_2] . A_v b_v[j_2] \tilde{s}_v[iN + k - m - l] \Big) \Bigg] \Bigg) + N \frac{\sigma^2}{2}$$

$$\sigma^2_{y_u^R|\mathbf{B}[i]}[i] = \frac{1}{4} \Bigg[\sum_{\substack{k_1, m_1, l_1, v_1, \\ k_2, m_2, l_2, v_2}} \tilde{C}_u f_1 f_2 + \tilde{C}_u^* f_1^* f_2^* + C_u f_1 f_2^* + C_u^* f_1^* f_2 \Bigg] + N \frac{\sigma^2}{2}$$

where $C_u = C[m_1, m_2][j_{11}, j_{21}]$, $\tilde{C}_u = \tilde{C}[m_1, m_2][j_{11}, j_{21}]$,

$$f_1 = f_u[k_1, m_1, l_1, v_1], \; f_2 = f_u[k_2, m_2, l_2, v_2],$$
$$f_u[k, m, l, v] = s_{uk} z_v[l][j_2] A_v b_v[j_2] \tilde{s}_v[iN + k - m - l] \tag{24}$$

Note that $f_u[k, m, l, v]$ of individual prefiltering is different from $f_u[k, m, l, v]$ of joint prefiltering. The cost function that is to be minimized to determine the prefilter coefficients ($\mathbf{z}_u[.][n]$) can be formulated, by using the above derived conditional mean and conditional variance as follows

$$P^u_{E|\mathbf{B}[i]}[i] = Q \left(\frac{b_u[i] \mu_{y_u^R|\mathbf{B}[i]}[i]}{\sigma_{y_u^R|\mathbf{B}[i]}[i]} \right)$$

Now the prefilter of length L_z^u at time instant $i + 1$ for user u are calculated using stochastic gradient search method as follows

$$\mathbf{z}_u[.][i+1] = \mathbf{z}_u[.][i] - \mu \frac{\partial P_E^u}{\partial \mathbf{z}_u[.][i]}, \qquad u \in \{1, 2, \ldots, U\} \tag{25}$$

B. *MMSE based prefiltering*

By following the same arguments as that of Section IV. B and Section V. A the MMSE cost function for user u can be derived as

$$\xi^2_{u|\mathbf{B}[i]}[i] = \sum_{u=1}^{U} \Bigg(\frac{1}{4} \Bigg[\sum_{\substack{k_1, m_1, l_1, u_1, \\ k_2, m_2, l_2, u_2}} \tilde{R}_u f_1 f_2 + \tilde{R}_u^* f_1^* f_2^* + R_u f_1 f_2^* + R_u^* f_1^* f_2 \Bigg] + 1 - 2 b_u \mu_{y_u^R|\mathbf{B}[i]}[i] \Bigg)$$

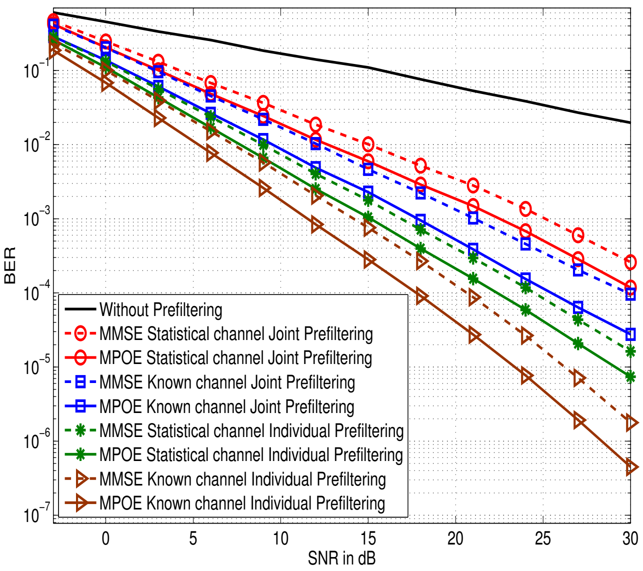

Fig. 3. Performance of prefilters for various SNRs (U=4, $\rho = 0.1$)

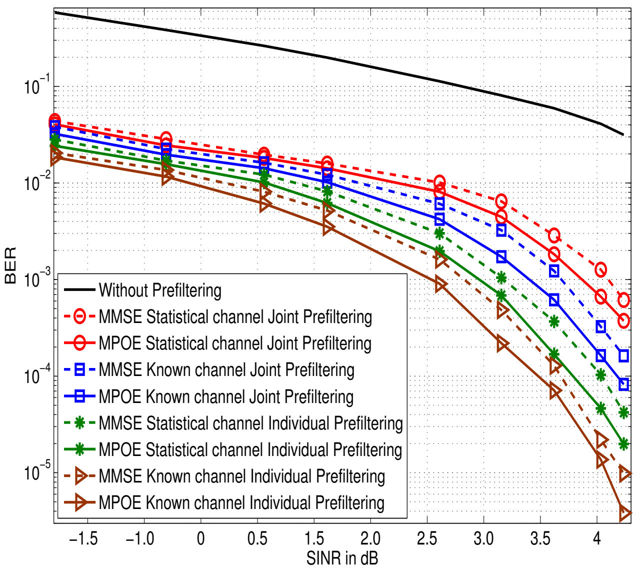

Fig. 4. Performance of prefilters for various SINRs (U=4, $\rho = 0.1$)

where f_1, f_2 are given in (24) and $\tilde{R}_u = \tilde{R}_u[m_1, m_2][j_{11}, j_{21}]$, $R_u = R_u[m_1, m_2][j_{11}, j_{21}]$. We follow a similar stochastic gradient descent approach as in (25) to compute $\mathbf{z}_u[.][i]$ by minimizing $\xi^2_{u|\mathbf{B}[i]}[i]$.

VI. SIMULATION RESULTS

Extensive simulations were carried out to calculate the prefilter coefficients and the corresponding bit error rate for various SNRs for both MPOE and MMSE based prefilters. BPSK constellation for bits was assumed with equal probability for bits +1 and -1. The processing gain N was assumed to be 128 and the number of users was taken to be 4. Channel was assumed to be complex Gaussian with both real and imaginary parts following an $i.i.d$ Gaussian distribution with $\sigma = 0.1655$ and $\mu = 0.5$. Channel length L_h^u was taken to be 3 and the prefilter length $L_z = 5$. Correlated spreading codes with correlation coefficient of $\rho = 0.1$ was considered.

A. BER performance of Joint and Individual Prefiltering

BER was calculated for 10,000 bits of input symbols for each channel and BERs of 100 such channels were averaged for each SNR. All the users were assumed to transmit with equal amplitude. SINR (Signal to Interference and Noise Ratio) was calculated at each SNR (SNR varies from 6 dB to 24 dB to calculate SINR). We have compared statistical channel model prefiltering with known channel model prefiltering proposed in [13]. BER performance for various SNRs and SINRs (signal to interference ratio) are shown in Fig. 3 and Fig. 4. respectively. From Fig. 3 and Fig. 4 it is clear that MPOE algorithms perform significantly better than the respective MMSE algorithms. We also observe that individual prefiltering performs much better than joint prefiltering. Furthermore, from Fig. 3 and Fig. 4, it is clear that the BER obtained using statistical channel model is close to BER obtained using the prefilter based on complete channel knowledge. In fact at low SNR the difference in performance is very small. However, as the SNR increases (noise variance decreasing) the channel variance term starts dominating the probability of error expression, consequently, the difference in performance increases at higher SNRs.

We have also analysed the performance of the proposed models with varying ρ. We varied the value of ρ from 0.01 to 0.7. Fig. 5 shows the BER performance for various ρ and a fixed SNR for both joint and individual prefiltering models. From Fig. 5, we observe that, as ρ increases, the BER increases due to increasing MAI.

B. Convergence Study

To examine the convergence of the proposed algorithm, we started with random values as prefilter coefficients in the gradient search algorithm. Based on the received signal energy the step size parameter (μ) was chosen as 0.01. The probability of error and mean square error were calculated at every iteration of the gradient search algorithm. The convergence curves for the MPOE and MMSE algorithms are shown in Fig. 6. Since individual prefilters perform better than joint prefilters, convergence performance was analysed only for individual prefilter case. We observe that after the algorithms converge, the residual error in MMSE algorithm is considerably higher than MPOE algorithm.

VII. CONCLUSION

We have developed MPOE based prefiltering algorithms based only on the statistical channel model. Simulation results

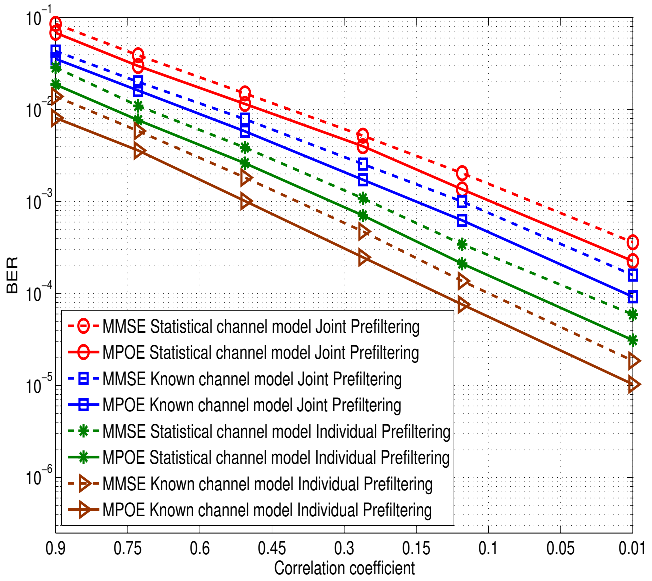

Fig. 5. BER performance for various ρ (U=4, SNR=20 dB)

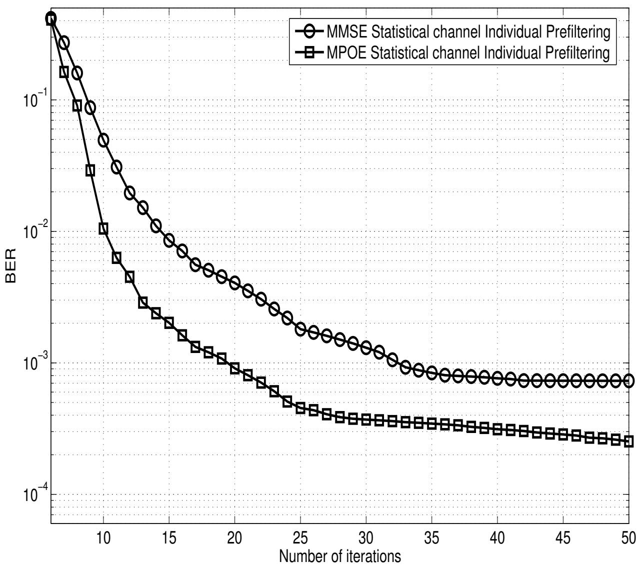

Fig. 6. Convergence curves for MPOE and MMSE individual prefiltering (U=4, SNR=20 dB, $\rho = 0.1$)

show that the BER performances are better than prefiltering algorithms designed based on MMSE criterion.

ACKNOWLEDGMENT

This work was supported in part by Department of Science and Technology, Government of India under the project title “Technology for cooperative utilization of open spectrum and precoding based MIMO systems”. The first author is funded by Microsoft Research India PhD fellowship award.

REFERENCES

[1] Neji Youssef, Cheng-Xiang Wang and Matthias Patzold, “A Study on the Second Order Statistics of Nakagami-Hoyt Mobile Fading Channels”, *IEEE Trans. Veh. Technol*, vol. 54, no. 4, pp. 1259-1265, July. 2005.

[2] Yahong Rosa Zheng and Chengshan Xiao, “Simulation Models With Correct Statistical Properties for Rayleigh Fading Channels”, *IEEE Trans. on Commun*, vol. 51, no. 6, pp. 920-928, June. 2003.

[3] Aditya Dua and Uday B. Desai, “MPOE based adaptive space-time multiuser detector for multipath CDMA channels”, *Proc. IEEE International Conference on Acoustics, Speech, and Signal Processing, 2002, ICASSP’02*, Orlando, Florida, 13-17 May 2002, vol. 3, pp. 2593-2596.

[4] Aditya Dua, Uday B. Desai and Ranjan K. Mallik, “Minimum probability of error based methods for adaptive multiuser detection in multipath DS-CDMA channels”, *IEEE Trans. on Wireless Communications*, vol. 3, no. 3, pp. 939-948, May. 2004.

[5] B. R Vojcic and W. M Jang, “Transmitter precoding in synchronous multiuser communications”, *IEEE Trans. on Communications*, vol. 46, no. 10, pp. 1346-1355, Oct. 1998.

[6] D. Reynolds, A. Host-Madsen and X. Wang, “Transmitter precoding for CDMA in fading multipath channels: strategy and analysis”, *Proc. IEEE International Conference on Communications, ICC-2003*, Alaska, USA, May. 11-15, 2003.

[7] Georgoulis S. L and Cruickshank D. G. M, “Transmitter-based inverse filters for reducing MAI and ISI in CDMA-TDD downlink”, *IEEE Trans. on Wireless Communications*, vol. 3, no. 2, Mar. 2004, pp. 353-358.

[8] Reynolds D, Xiaodong Wang and Modi K. N, “Interference suppression and diversity exploitation for multiantenna CDMA with ultra-low complexity receivers”, *IEEE Trans. on Signal Processing*, vol. 53, no. 8, part 2, Aug. 2005, pp. 3226-3237.

[9] Yuhang Wang, Xian-Da Zhang and He Huang, “Minimum probability of error-based adaptive multiuser decision feedback detection”, *IEEE Signal Processing Letters*, vol. 13, no. 5, pp. 257-260, May. 2006.

[10] Chen S, Mulgrew B, Chang E. S and Gibson G. J, “Space translation properties and the minimum-BER linear-combiner DFE”, *Proc. IEE Communications*, vol. 145, no. 5, pp. 316-322, Oct. 1998.

[11] Y Ding, T. N Davidson, Z. Q Luo and K. M Wong, “Minimum BER block precoders for zero-forcing equalization”, *IEEE Trans. on Signal Processing*, vol. 51, no. 9, pp. 2410-2423, Sep. 2003.

[12] Palomar D. P, Bengtsson M and Ottersten B, “Minimum BER linear transceivers for MIMO channels via primal decomposition”, *IEEE Trans. on Signal Processing*, vol. 53, no. 8, part 1, pp. 2866-2882, Aug. 2005.

[13] Mohit Garg, G. Kannan, S. N. Merchant and U. B. Desai, “MPOE based prefiltering for DS-CDMA Systems”, *Proc. 2006 IEEE Mountain Workshop on Adaptive and Learning Systems SMCals/06*, Logan, U.S.A, Jul. 24-26, 2006.

[14] H. Sampath and A. Paulraj, “Linear precoding for space-time coded systems with known fading correlations”, *IEEE Commun. Letters*, vol. 6, no. 6, pp. 239-241, Jun. 2004.

[15] June Chul Roh and Baskar D. Rao, “Multiple antenna channels with partial channel state information at the transmitter”, *IEEE Trans. Wireless Communications*, vol. 3, no. 2, pp. 677-688, Mar. 2004.

[16] Hamid Reza Bahrami and Thos Le-Ngoc, “MMSE-based MIMO precoder using partial channel information”, *Proc. IEEE International Conference on Wireless and Mobile Computing, Networking and Communications, 2005, WiMob’2005*, Montreal, Canada, 22-24 Aug. 2005, vol. 1, pp. 211-216

[17] Jane W. Huang, Edward K. S. Au and Vincent K. N. Lau, “Linear Precoding for Space-Time Coded MIMO Systems using Partial Channel State Information”, *2006 IEEE International Symposium on Information Theory*, Seatle, Washington, USA, 9-14 Jul. 2006, pp. 391-396

[18] Athanasios P. Liavas, “Tomlinson-Harashima precoding with partial channel knowledge”, *IEEE Trans. Communications*, vol. 53, no. 1, pp. 5-9, Jan. 2005.

[19] S. Moshavi, “Multi-user detection for DS-CDMA communications”, *IEEE Communications Magazine*, vol. 34, no. 10, pp. 124-136, oct 1996.

[20] Mohit Garg, “Multiuser signal processing techniques for DS-CDMA communication systems”, *MTech dissertation*, IIT Bombay, 2005, http://users.vectorstar.net/∼ mohitgarg/learn/MohitGargDDPThesis.pdf.